International Journal of Science and Engineering Science Research
Volume 2 | Issue 2 | April-June 2026 | ISSN 3049-2793
Peer Reviewed (Referred Journal)
DOI: https://doi.org/10.63671/ijsesr.v2i2.115

Comprehensive Evaluation of the Leaf spring Manufacturing station: Assessing Efficiency and Reliability

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Abstract

An essential and crucial component of the suspension system is the leaf spring. Finding an ideal design that works well under a range of loading circumstances is the aim of this investigation. This study offers a thorough availability and performance modelling evaluation of the leaf spring production sector, emphasizing the identification of crucial subsystems, the assessment of failure behaviours, and the measurement of their effect on total productivity. Here, we look at three different component conditions: fine, diminished, and crashed. The breakdown and maintenance rates of each component are assumed to be fixed and substantially separate. The system was analytically formulated using the Markov Birth-Death method. To compute the stable state probability, the Chapman-Kolmogorov differential equations were established based on the system's state-transition relationships, considering a spectrum of failure and repair rates. Decision matrices are derived from different performance levels in terms of availability. The model's practical application is illustrated by numerical findings and case studies, which provide insights into scheduling preventative maintenance, improving production reliability, and allocating resources optimally. Therefore, the results of this study are considered to be helpful in analyzing availability and identifying the best maintenance techniques that may be used going forward to improve system performance.

Keywords

modelling, Markov Birth-Death process, the Chapman-Kolmogorov differential equations, availability analysis, Runge-Kutta Method.

1. Introduction

In the very competitive automotive sector of today, the demand for reliable, efficient, and cost-effective suspension components has increased significantly. Leaf springs, being a vital part of heavy and light vehicle suspension systems, directly influence ride comfort, load-bearing capacity, and vehicle safety. The production of leaf springs is a multi-stage, intricate procedure such as heat treatment, shot peening, assembly, and quality inspection. Each stage is susceptible to machine breakdowns, process delays, and variability in operational performance, which collectively affect the overall productivity and availability of the system. Performance modeling and availability analysis have emerged as effective approaches for

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understanding, evaluating, and improving manufacturing systems. In the context of the leaf spring manufacturing sector, These evaluations offer insightful information about the trade-offs between maintenance planning, downtime, and system performance.

Despite its industrial importance, limited research has been directed toward the performance modeling of leaf spring manufacturing systems. Most existing studies in reliability and performance analysis focus on general production systems, whereas domain-specific investigations are essential to capture the unique challenges of this sector. Hence, there is a pressing need to develop systematic models that evaluate the performance and availability of leaf spring production lines, considering real-world constraints and stochastic variations. A Markov modeling and dependability analysis of a fertilizer plant's urea synthesis system was presented by Aggarwal, Kumar, and Singh (2015). Bansal & Tyagi (2018) used the orthogonal matrix method to analyze the screw manufacturing plant's reliability. In 2024, Burnett, Gottlieb, and Grant assessed the performance and stability analysis of additive mixed-precision runge-kutta methods. Ram and Goyal (2017) applied **stochastic modeling** to assess the performance of a wind-powered power plant, while Gupta, Lal, Sharma, and Singh (2007) investigated the **reliability and availability of serial processes** in a plastic-pipe producing plant. Gupta, Ekata, and Batra (2019) used neural network architecture to estimate reliability metrics in biofuel plant subtilizing optimization with particle swarms. A system for decision-making for optimal availability of sequential parallel systems was presented by Kumar and Garg in 2023, Using a case study of PSO. Kumar (2018) selected performance indicators and assessed the efficiency of a globally impaired manufacturing system.Kumar, Ghosh and Banik (2021) investigated numerically the intermittent reaction of limited bulk arriving or service lines with multiple operating periods. Mechanisms and consequences of market penetration for autonomous vehicles: A Markov prediction method's insights were examined by Zhang, Yao, Xiao, Zhang, and Cai (2024). Performability and maintenance for a subcritical thermic power plant's coal ash handling system were determined by Malik & Tewari (2023). Mehta, Singh, and Singh (2017) assessed a series-parallel system's availability and reliability in the event of a random failure. Malik and Tewari (2018) discussed the efficacy forecasting and preservation priority selection of the water discharge equipment of a coal-generated thermal power plant. According to Mohamed, Islam, Ragab, and Eman (2018), a thorough model of unreliability, inaccessibility, and keep in (RAM) for commercial enterprise equipment assessments was proposed. Weik and Nieben (2017) took a An almost life-or-death procedure for modeling railway systems' combined capacity and dependability. According to Reena and Basotia (2020), the moulding and unavailability of cement fabricating plants susceptible to human failure and coverage variables were investigated. To solve autonomous differential equations, Salihu & Lawan (2025) compared the 4th-order and 6th-stage 5th-order Runge-Kutta methods. Tyagi, Bansal, Agarwal, and Yadav (2021) offered mathematical sculpturing and accessibility investigating of a blade spring constructing factory. In their evaluation of cyber-physical system (CPS) reliability modeling, Yang, Wang, Wen, and Xu (2021) took communication failures into account. Using a combination of the Laplace transform and the fourth-order Runge-Kutta method, Sahani et al. (2025) assessed the dependability of an industrial plywood production plant.

2. System Description

The manufacturing process involves multiple sequential and interdependent operations that transform raw materials (such as alloy steel plates) into finished leaf springs. The system can be conceptualized as a multi-stage production line, where the efficiency and availability of each stage directly influence the overall system performance. This system comprises of four subsystems.

(i) Raw Material Preparation (A): Cutting, cleaning, and inspection of steel plates to ensure dimensional accuracy and quality. Two machines are operating for this purpose.

(ii) Forming and Heat Treatment (B): Processes such as cambering, hardening, tempering, and shot peening, which impart strength and elasticity to the leaf springs. There are also two machines operating.

(iii) Surface Treatment and Finishing (C): Operations like painting, coating, and polishing to enhance durability and corrosion resistance. A single unit provided for this purpose.

(iv) Assembly and Inspection (D): Assembly of individual leaves into spring packs, followed by quality inspection and load testing. Here, also single unit provided.

3. Assumptions

(i) Each subsystem is subject to random failures.
(ii) An exponential distribution of failure times suggests a steady failure.
(iii) It is also considered that repair rates are fixed and that repair times are exponentially dispersed.
(iv) Only one type of failure per subsystem is considered, without differentiating between minor and major breakdowns.
(v) Sufficient repair facilities and spare parts are available, so repairs begin immediately after failure.
(vi) Environmental factors like temperature or humidity do not significantly affect system reliability.

4. Notations

S_0	:	All subsystems are working.
S_1	:	System is working with reduced efficiency since one part has been failed of subsystem A.
S_2	:	The system is operating less efficiently since one unit has been failed of subsystem B.
S_3	:	The efficiency of the system is decreased since two unit has been failed of subsystem A &B.
S_4, S_6, S_8, S_{13}	:	System is failed due to subsystem C has been failed which presents by states S_4, S_6, S_8, S_{13}.
S_5, S_7, S_9, S_{12}	:	System is failed due to subsystem D has been failed which presents by states S_5, S_7, S_9, S_{12}.
S_{11}, S_{14}	:	System is failed in statesS_{11}, S_{14} due to subsystem B has been failed in these states.
S_{10}, S_{15}	:	System is failed in statesS_{10}, S_{15} due to subsystem B has been failed in these states.
α_1, β_1	:	Continual failure and repair rate of the states S_{10}, S_{15}
α_2, β_2	:	Changeless failure and repair rate of the states S_{11}, S_{14}
α_3, β_3	:	Constant failure and repair rate of states S_4, S_6, S_8, S_{13}

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α_4, β_4	:	Constant failure and repair rate of states S_4, S_6, S_8, S_{13}
α_5, β_5	:	Constant failure and repair rate of the states S_1, S_3
α_6, β_6	:	Constant failure and repair rate of the states S_2, S_3

5. The mathematical formulation of the system.

The system's behavior is mathematically described by the Chapman-Kolmogorov equations, which are derived from basic probabilistic assumptions for a Markov birth-death process The mnemonic rule is used to determine the coefficients. The time evolution of the probability for each state is determined by considering the net balance of incoming and outgoing transition rates, where the rate of change for any state's occupation probability equals the summed inflows from all other states minus the summed outflows to all other states. This foundational principle governs both the transient and steady-state behaviors of such systems and thus forms the basis for deriving their respective mathematical.

6. Transient state.

By applying the general birth and death technique to each state, a system of differential equations is obtained.

$$P_0'(t) + (\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6)P_0 = \beta_5 P_1 + \beta_6 P_2 + \beta_3 P_4 + \beta_4 P_5 \quad (1)$$

$$P_1'(t) + (\alpha_1 + \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6)P_1 = \alpha_5 P_0 + \beta_1 P_{10} + \beta_6 P_3 + \beta_3 P_6 + \beta_4 P_7 \quad (2)$$

$$P_2'(t) + (\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \beta_6)P_2 = \alpha_6 P_0 + \beta_2 P_{11} + \beta_5 P_3 + \beta_3 P_8 + \beta_4 P_9 \quad (3)$$

$$P_3'(t) + (\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \beta_5 + \beta_6)P_3 = \alpha_5 P_2 + \alpha_6 P_1 + \beta_1 P_{15} + \beta_2 P_{14} + \beta_3 P_{13} + \beta_4 P_{12} \quad (4)$$

Figure-1

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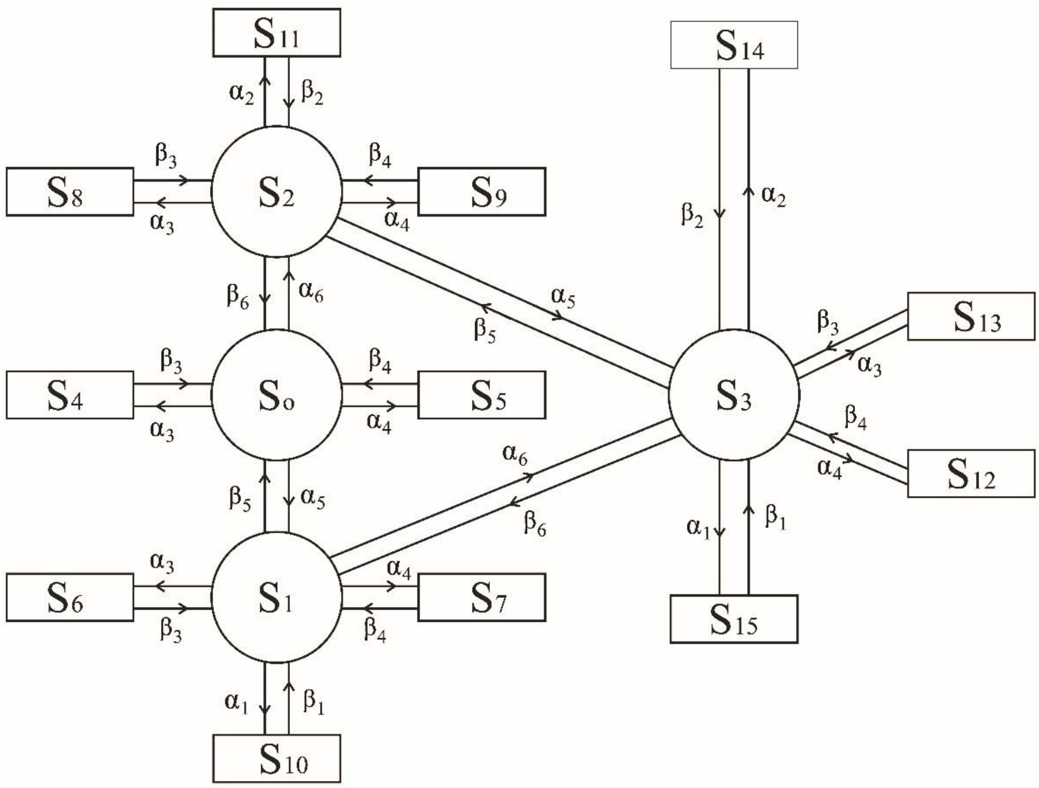

Similarly, Differential equation for other states is given as

$$P_j'(t) + \beta_i P_j(t) = \alpha_i P_k(t) \quad (5)$$

Where

$(i = 3,4 \;\&\; j = 4,5 \quad When\ k = 0)$

$(i = 3,4,1 \;\&\; j = 6,7,10 \quad When\ k = 1)$

$(i = 3,4,2 \;\&\; j = 8,9,11 \quad When\ k = 2)$

$(i = 4,3,2,1 \;\&\; j = 12,13,14,15 \quad When\ k = 3)$

Numerical simulations were conducted for various sub-system failure and repair rates, The system of differential equations (1)-(5), along with the initial conditions, was integrated over a time interval of t=0 to t=360 days utilizing the fourth-order Runge-Kutta method.Using the mnemonic rule, we created a set of Chapman-Kolmogorov differential equations for the system's dependability, which are provided by,

$$P_j'(t) = P_j(t) * Q \quad (6)$$

Where Q=

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$$\begin{bmatrix}
-X_1 & \beta_5 & \beta_6 & 0 & \beta_3 & \beta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_5 & -X_2 & 0 & \beta_6 & 0 & 0 & \beta_3 & \beta_4 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 \\
\alpha_6 & 0 & -X_3 & \beta_5 & 0 & 0 & 0 & 0 & \beta_3 & \beta_4 & 0 & \beta_2 & 0 & 0 & 0 & 0 \\
0 & \alpha_6 & \alpha_5 & -X_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \beta_4 & \beta_3 & \beta_2 & \beta_1 \\
\alpha_3 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\alpha_4 & 0 & 0 & 0 & 0 & -\beta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_3 & 0 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_4 & 0 & 0 & 0 & 0 & 0 & -\beta_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha_3 & 0 & 0 & 0 & 0 & 0 & -\beta_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha_4 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_4 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \alpha_4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_4 & 0 & 0 & 0 \\
0 & 0 & 0 & \alpha_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_3 & 0 & 0 \\
0 & 0 & 0 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_2 & 0 \\
0 & 0 & 0 & \alpha_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\beta_1
\end{bmatrix}$$

The sequence of operations moves from P_i to P_{i+1} as

$$P_{i+1} = P_i + h(k_1 + 2k_2 + 2k_3 + k_4)/6 \qquad (7)$$

Where, $k_1 = P_i * Q$, $k_2 = (P_i + hk_1/2) * Q$,

$k_3 = (P_i + hk_2/2) * Q$, $k_4 = (P_i + hk_3) * Q$

Consequently, we can determine all the probability $P_0(t), P_1(t)$, ...$P_{15}(t)$. The system's reliability R(t) is the total of its reliabilities in both its reduced state and full capacity, i.e.,

$$R(t) = P_0(t) + P_1(t) + P_2(t) + P_3(t) \qquad (8)$$

7. Steady state

The following limitations are used such that ascertain the system's steady state probabilities: $\frac{d}{dt} \to 0$, as $t \to \infty$. Therefore, setting the derivative of all probabilities to zero yields the system's long-term availability A (∞). Using this, equations (1) through (5) can be simplified as

$$X_1 P_0 = \beta_5 P_1 + \beta_6 P_2 + \beta_3 P_4 + \beta_4 P_5 \quad , X_1 = \alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 \qquad (9)$$

$$X_2 P_1 = \alpha_5 P_0 + \beta_1 P_{10} + \beta_6 P_3 + \beta_3 P_6 + \beta_4 P_7 \quad , X_2 = \beta_5 + \alpha_1 + \alpha_3 + \alpha_4 + \alpha_6 \qquad (10)$$

$$X_3 P_2 = \alpha_6 P_0 + \beta_2 P_{11} + \beta_5 P_3 + \beta_3 P_8 + \beta_4 P_9 \quad , X_3 = \alpha_2 + \alpha_3 + \alpha_4 + \alpha_5 + \beta_6 \qquad (11)$$

$$X_4 P_3 = \alpha_5 P_2 + \alpha_6 P_1 + \beta_1 P_{15} + \beta_2 P_{14} + \beta_3 P_{13} + \beta_4 P_{12} \qquad 12)$$

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Where $X_4 = \alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \beta_5 + \beta_6$

Similarly,

Differential equation for other states is given as

$$\beta_i P_j(t) = \alpha_i P_k(t) \quad (13)$$

Solving these equations iteratively and Using Normalizing condition, $\sum_{j=0}^{15} P_j = 1$ and Initial condition, $P_i(t) = 1$, $i = 0,1,2,3$ and other $P_i(t) = 0$ We obtain the likelihood of reaching full functioning capability P_0 as,

$$P_0 = \frac{1}{4}\left[1 + \frac{\alpha_1}{2\,\beta_1} + \frac{\alpha_2}{2\,\beta_2} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4}\right]^{-1} \quad (14)$$

The measure of system availability can be derived by summing the probabilities of all fully functioning and partially operational scenarios. i.e.

$$A(\infty) = P_0 + P_1 + P_2 + P_3$$

$$A(\infty) = \left[1 + \frac{\alpha_1}{2\,\beta_1} + \frac{\alpha_2}{2\,\beta_2} + \frac{\alpha_3}{\beta_3} + \frac{\alpha_4}{\beta_4}\right]^{-1} \quad (15)$$

8. Results and calculations for the transient state

The system's dependability was quantified using Equation (8), with estimates derived from varying repair and failure rate combinations. The scope of the numerical analysis was limited to the primary subsystems. Matrix calculus and the Runge-Kutta method were employed to compute the system's reliability based on specific aggregated breakdown and recovery rates. The Mean Time Between Failures (MTBF), obtained through Simpson's one-third rule, is reported in days in the final row of the failure rate tables. The following tables present a detailed analysis of how the failure and repair rates of various components influence system availability

8.1. Influence of failure rate α_1 on system reliability

We investigated the effect of the failure rate on system dependability through the alteration of these values as: $\alpha_1 = 0.001,\ 0.002,\ 0.003,\ 0.004$. Other failure and repair rate are kept constant as $\alpha_2 = 0.003$, $\alpha_3 = 0.005$, $\alpha_4 = 0.007$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_1 = 0.02$ $\beta_2 = 0.04$, $\beta_3 = 0.06$, $\beta_4 = 0.08$, $\beta_5 = 0.10$, $\beta_6 = 0.12$. These statistics were used to calculate the system's dependability; the results are displayed in Table 1.This array demonstrates that as time increases, the system's reliability drops by 0.76 percent. But when the failure rate fluctuates between 0.001 and 0.004, it drops by about 5.7 percent, and MTBF drops by 5.7 percent as well.

Table-I

$\alpha_1\rightarrow$ Days↓	0.001	0.002	0.003	0.004
30	0.8102	0.7941	0.7787	0.7639
60	0.8096	0.7935	0.7781	0.7633
90	0.8090	0.7930	0.7776	0.7628
120	0.8085	0.7925	0.7770	0.7623

150	0.8079	0.7919	0.7765	0.7617
180	0.8074	0.7914	0.7759	0.7612
210	0.8068	0.7908	0.7754	0.7607
240	0.8063	0.7903	0.7749	0.7602
270	0.8057	0.7898	0.7744	0.7596
300	0.8052	0.7892	0.7738	0.7591
330	0.8047	0.7887	0.7733	0.7586
360	0.8041	0.7881	0.7728	0.7581
MTBF	258.31	253.18	248.25	243.53

8.2. Effect of failure rate α_2 on Reliability of the system

In this, We've investigated the changing of failure rate on reliability of the system by Choosing $\alpha_2 = 0.003, 0.004, 0.005, 0.006$ another parameter as $\alpha_1 = 0.001$, $\alpha_3 = 0.005$, $\alpha_4 = 0.007$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_1 = 0.02$ $\beta_2 = 0.04$, $\beta_3 = 0.06$, $\beta_4 = 0.08$, $\beta_5 = 0.10$, $\beta_6 = 0.12$. The system's dependability has been evaluated using these data, and Table II displays the findings of this study. The array makes it evident that dependability and MTBF approximately decrease by 2.95% every time the failure rate increases from 0.003 to 0.006.The dependability crimped by around 0.75 percent between 30 and 360 days.

Table-II

α_2→ Days↓	0.003	0.004	0.005	0.006
30	0.8101	0.8021	0.7941	0.7863
60	0.8097	0.8016	0.7936	0.7858
90	0.8090	0.8010	0.7930	0.7852
120	0.8085	0.8005	0.7925	0.7847
150	0.8079	0.7999	0.7919	0.7842
180	0.8074	0.7993	0.7914	0.7836
210	0.8068	0.7988	0.7908	0.7831
240	0.8063	0.7983	0.7902	0.7825
270	0.8056	0.7977	0.7898	0.7820
300	0.8052	0.7972	0.7892	0.7815
330	0.8047	0.7966	0.7888	0.7809
360	0.8041	0.7961	0.7882	0.7804
MTBF	258.31	255.74	253.19	250.70

8.3. Effect of failure rate α_3 on system reliability.

For evaluated the influence of failure rate we choose $\alpha_3 = 0.005, 0.006, 0.007, 0.008$ another parameter as $\alpha_1 = 0.001$, $\alpha_2 = 0.003$, $\alpha_4 = 0.007$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_1 = 0.02$ $\beta_2 = 0.04$, $\beta_3 = 0.06$, $\beta_4 = 0.08$, $\beta_5 = 0.10$, $\beta_6 = 0.12$. The system's reliability is determined using these data, and the findings of this investigation are given in table III. According to this table, reliability and MTBF roughly drop by 4.0% while the failure rate movement from 0.005 to 0.008. Reliability drops from 0.74 to 0.94 percent as time grows from 30 to 360 days.

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Table-III

α_3→ Days↓	0.005	0.006	0.007	0.008
30	0.8101	0.7994	0.7889	0.7785
60	0.8097	0.7988	0.7882	0.7778
90	0.8090	0.7982	0.7876	0.7771
120	0.8085	0.7976	0.7869	0.7764
150	0.8079	0.7970	0.7863	0.7758
180	0.8074	0.7964	0.7857	0.7751
210	0.8068	0.7958	0.7850	0.7747
240	0.8063	0.7952	0.7843	0.7738
270	0.8056	0.7946	0.7838	0.7731
300	0.8052	0.7940	0.7832	0.7725
330	0.8047	0.7934	0.7825	0.7718
360	0.8041	0.7929	0.7819	0.7712
MTBF	258.31	254.78	251.34	247.97

8.4. Influence of failure rate α_4 on Reliability of the system

For investigation the effect on the system's dependability we choose failure rate $\alpha_4 = 0.007, 0.008, 0.009, 0.010$ Other failure and repair rate fixed constant as $\alpha_1 = 0.001$, $\alpha_2 = 0.003$, $\alpha_3 = 0.005$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_1 = 0.02$ $\beta_2 = 0.04$, $\beta_3 = 0.06$, $\beta_4 = 0.08$, $\beta_5 = 0.10$, $\beta_6 = 0.12$. Table IV provides the outcomes of the reliability and MTBF assessments conducted using the provided data. The findings demonstrate that raising the failure rate from 0.007 to 0.010 causes a 3.0% drop in both the system's reliability and its MTBF. Additionally, the system's reliability decreases from 0.76% to 0.94% when the operational duration is prolonged from 30 to 360 days.

Table-IV

α_4→ Days↓	0.007	0.008	0.009	0.010
30	0.8102	0.8020	0.7940	0.7862
60	0.8096	0.8015	0.7934	0.7855
90	0.8090	0.8008	0.7928	0.7848
120	0.8085	0.8002	0.7921	0.7842
150	0.8079	0.7997	0.7915	0.7835
180	0.8074	0.7991	0.7908	0.7828
210	0.8068	0.7985	0.7902	0.7821
240	0.8063	0.7979	0.7896	0.7815
270	0.8056	0.7973	0.7889	0.7807
300	0.8052	0.7967	0.7883	0.7801
330	0.8047	0.7961	0.7877	0.7795
360	0.8041	0.7955	0.7871	0.7788
MTBF	258.31	255.64	253.00	250.43

8.5. Effect of Repair rate β_1 on Reliability of the system

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the impact of repair range β_1 on system dependability is investigated By altering their values as $\beta_1 = 0.01, 0.03, 0.05, 0.07$ Other failure and repair rate fixed constant as $\alpha_1 = 0.001$, $\alpha_2 = 0.003$, $\alpha_3 = 0.005$, $\alpha_4 = .004$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_2 = 0.04$, $\beta_3 = 0.06$, $\beta_4 = 0.08$, $\beta_5 = 0.10$, $\beta_6 = 0.12$. These statistics are utilized to determine the system's dependability and the outcome are displayed in table V. This table demonstrates that as time increases, the system's reliability drops by 0.75 percent. But when the failure rate fluctuates between 0.01 and 0.07, it rises by about 3.5 percent, and MTBF rises by 3.5 percent as well.

Table-V

$\beta_1 \rightarrow$ Days↓	0.01	0.03	0.05	0.07
30	0.7941	0.8157	0.8202	0.8222
60	0.7936	0.8151	0.8196	0.8216
90	0.7930	0.8146	0.8190	0.8211
120	0.7925	0.8140	0.8185	0.8205
150	0.7919	0.8135	0.8179	0.8199
180	0.7914	0.8129	0.8173	0.8194
210	0.7908	0.8123	0.8168	0.8188
240	0.7903	0.8118	0.8163	0.8182
270	0.7898	0.8112	0.8157	0.8177
300	0.7892	0.8107	0.8151	0.8171
330	0.7887	0.8101	0.8146	0.8166
360	0.7882	0.8096	0.8140	0.8160
MTBF	253.19	260.07	261.49	262.14

8.6. Effect of Repair rate β_2 on Reliability of the system

In order to evaluate the impact of repair rate β_2 on system reliability, their values are adjusted as $\beta_2 = 0.03, 0.05, 0.07, 0.09$ Other failure and repair rate fixed constant as $\alpha_1 = 0.001$, $\alpha_2 = 0.003$, $\alpha_3 = 0.005$, $\alpha_4 = .004$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_1 = 0.02$, $\beta_3 = 0.06$, $\beta_4 = 0.08$, $\beta_5 = 0.10$, $\beta_6 = 0.12$.Table VI shows the results of calculating the system's reliability using these statistics.This table demonstrates that as time increases, the system's reliability drops by 0.75 percent. But when the failure rate fluctuates between 0.03 and 0.09, it rises by about 2.74 percent, and MTBF rises by 2.74 percent as well.

Table-VI

$\beta_2 \rightarrow$ Days↓	0.03	0.05	0.07	0.09
30	0.8021	0.8152	0.8210	0.8241
60	0.8016	0.8147	0.8204	0.8235
90	0.8010	0.8141	0.8198	0.8230
120	0.8005	0.8135	0.8193	0.8224
150	0.7999	0.8130	0.8187	0.8218
180	0.7993	0.8124	0.8182	0.8213
210	0.7988	0.8118	0.8176	0.8207

240	0.7983	0.8113	0.8170	0.8201
270	0.7977	0.8107	0.8165	0.8196
300	0.7971	0.8102	0.8159	0.8190
330	0.7966	0.8096	0.8154	0.8185
360	0.7961	0.8091	0.8148	0.8179
MTBF	255.73	259.91	261.18	262.74

8.7. Effect of Repair rate β_3 on system Reliability.

Influence of repair rate β_3 on system reliability is explored by modifying its values as $\beta_3 = 0.05, 0.07, 0.09, 0.11$ Other failure and repair rate choose as $\alpha_1 = 0.001$, $\alpha_2 = 0.003$, $\alpha_3 = 0.005$, $\alpha_4 = .004$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_1 = 0.02$, $\beta_2 = 0.04$, $\beta_4 = 0.08$, $\beta_5 = 0.10$, $\beta_6 = 0.12$.Table VII provides the outcomes of the system dependability assessment based on these data. The failure rate is increased from 0.05 to 0.11 in this table, resulting in a 4.57 percent increase in reliability and MTBF. However, as time goes on, the system's reliability drops by 0.74 percent.

Table-VII

$\beta_3\rightarrow$ Days↓	0.05	0.07	0.09	0.11
30	0.7994	0.8181	0.8289	0.8359
60	0.7989	0.8176	0.8283	0.8353
90	0.7983	0.8170	0.8278	0.8347
120	0.7978	0.8164	0.8272	0.8342
150	0.7972	0.8158	0.8266	0.8336
180	0.7967	0.8153	0.8260	0.8330
210	0.7961	0.8147	0.8255	0.8325
240	0.7956	0.8142	0.8249	0.8319
270	0.7950	0.8136	0.8244	0.8313
300	0.7945	0.8131	0.8238	0.8308
330	0.7939	0.8125	0.8233	0.8302
360	0.7934	0.8120	0.8227	0.8297
MTBF	254.88	260.84	264.28	266.51

8.8. Effect of Repair rate β_4 on Reliability of the system

In order to study the affect of repair rate β_4 on system availability, their values are varied as $\beta_4 = 0.07, 0.09, 0.11, 0.13$ Other failure and repair rate choose as $\alpha_1 = 0.001$, $\alpha_2 = 0.003$, $\alpha_3 = 0.005$, $\alpha_4 = .004$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_1 = 0.02$, $\beta_2 = 0.04$, $\beta_3 = 0.06$, $\beta_5 = 0.10$, $\beta_6 = 0.12$. These statistics were used to calculate the system's reliability; the findings are displayed in Table VIII. This table demonstrates that raising the failure rate from 0.07 to 0.13 results in a 3.85% increase in reliability and MTBF. But as time passes, the system's dependability drops by 0.75 percent.

Table-VIII

$\beta_4\rightarrow$ Days↓	0.07	0.09	0.11	0.13

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Peer Reviewed (Referred Journal)
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30	0.8021	0.8167	0.8263	0.8330
60	0.8016	0.8162	0.8257	0.8324
90	0.8010	0.8156	0.8252	0.8319
120	0.8005	0.8150	0.8246	0.8313
150	0.7999	0.8145	0.8240	0.8307
180	0.7993	0.8139	0.8235	0.8301
210	0.7988	0.8133	0.8229	0.8296
240	0.7982	0.8128	0.8223	0.8290
270	0.7977	0.8122	0.8218	0.8285
300	0.7972	0.8168	0.8212	0.8279
330	0.7966	0.8111	0.8207	0.8274
360	0.7961	0.8106	0.8201	0.8268
MTBF	255.73	260.59	263.45	265.59

9. Calculations and findings for the steady state.

This section assesses the effects of several criteria on long-term accessibility. Additionally, we have examined the combinations of the system's governing factors that have the most effects on the system.

9.1. Impact of failure rate α_1 & α_2 on the system's long run availability.

Equation (15) is used to examine the impact of failure rates and availability over the long term, and their values are varied as $\alpha_1 = 0.001,\ 0.002,\ 0.003,\ 0.004$, $\alpha_2 = 0.003, 0.004, 0.005, 0.006$. Other failure and repair rate are constant as , $\alpha_3 = 0.005, \alpha_4 = .004$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_1 = 0.02$, $\beta_2 = 0.04$, $\beta_3 = 0.06$, $\beta_4 = 0.08$, $\beta_5 = 0.10$, $\beta_6 = 0.12$. The long-term dependability for several kinds of failure and repair rates is shown in Table IX. According to the table, failure rate α_1 has an approximate impact of 5.7 percent on long-term availability, whereas failure rate α_2 has an approximate impact of 2.95 to 2.78 percent.

Table-IX

$\alpha_1\rightarrow$ $\alpha_2\downarrow$	0.001	0.002	0.003	0.004
0.003	0.8108	0.7947	0.7792	0.7644
0.004	0.8026	0.7869	0.7717	0.7571
0.005	0.7947	0.7792	0.7644	0.7500
0.006	0.7869	0.7717	0.7571	0.7431

9.2. Effect of repair rate β_1 & β_2 on long run availability of the system.

The consequence of failure rate β_1 & β_2 on long run availability is studied by using equation (15) and varying their values as, $\beta_1 = 0.01, 0.03, 0.05, 0.07$, $\beta_2 = 0.03, 0.05, 0.07, 0.09$. Other failure and repair rate are constant as , $\alpha_1 = 0.001$, $\alpha_2 = 0.003, \alpha_3 = 0.005, \alpha_4 = .004$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_3 = 0.06$, $\beta_4 = 0.08$ $\beta_5 = 0.10$, $\beta_6 = 0.12$. Table IX displays the long-term availability for numerous arrangement of failure and repair rates. According to the table X, repair rate β_1 has an approximate impact of 3.48 to 3.58 percent on long-term availability, whereas repair rate β_2 has an approximate impact of 2.69 to 2.79 percent.

International Journal of Science and Engineering Science Research
Volume 2 | Issue 2 | April-June 2026 | ISSN 3049-2793
Peer Reviewed (Referred Journal)
DOI: https://doi.org/10.63671/ijsesr.v2i2.115

Table-X

$\beta_1 \rightarrow$ $\beta_2 \downarrow$	0.01	0.03	0.05	0.07
0.03	0.7869	0.8080	0.8125	0.8143
0.05	0.7995	0.8214	0.8259	0.8279
0.07	0.8050	0.8272	0.8318	0.8338
0.09	0.8081	0.8304	0.8351	0.8371

9.3. The affect of failure rate α_1 & repair rate β_2 on system's long term availability.

The result of failure rate α_1 & repair rate β_2 on long-term accessibility is studied by using equation (15) and differing principles as, $\alpha_1 = 0.001,\ 0.002,\ 0.003,\ 0.004$ $\beta_2 = 0.03, 0.05, 0.07, 0.09$. Other failure and repair rate are constant as, $\alpha_2 = 0.003$, $\alpha_3 = 0.005$, $\alpha_4 = .004$, $\alpha_5 = 0.009$, $\alpha_6 = 0.011$, $\beta_1 = .02$, $\beta_3 = 0.06$, $\beta_4 = 0.08$, $\beta_5 = 0.10$, $\beta_6 = 0.12$. The long-term availability for various combinations of repair and failure rates is detailed in Table X. According to the table, failure rate α_1 has an approximate impact of 5.68 to 5.82 percent on long-term availability, whereas repair rate β_2 has an approximate impact of 2.74 to 2.59 percent.

Table-XI

$\alpha_1 \rightarrow$ $\beta_2 \downarrow$	0.001	0.002	0.003	0.004
0.03	0.8027	0.7869	0.7717	0.7571
0.05	0.8158	0.7995	0.7838	0.7688
0.07	0.8216	0.8050	0.7891	0.7739
0.09	0.8247	0.8081	0.7921	0.7767

10. Analysis of results

The suggested approach is simple to employ in complicated systems with many differential equations and aids in calculating the system's MTBF, long-term inaccessibility, and dependability. The data in Tables 1 through XI are calculated to support the maintenance plan and establish the priority of repairs for the different subsystems.Sub-system B has the most significant impact on ensuring the system's continued functionality and dependability, according to a comparison of these tables. The system's long-term availability and dependability are likewise impacted by the other subsystems; however sub-system B has a greater impact. Subsystem B should therefore be given top priority because its failure and repair rates have a far greater outcome on unit availability than those of other subsystems. In figure 2-3, the impact of subsystem B's failure rate (α_1)and repair rate β_1 the system's dependability is also graphically displayed.

Figure-2

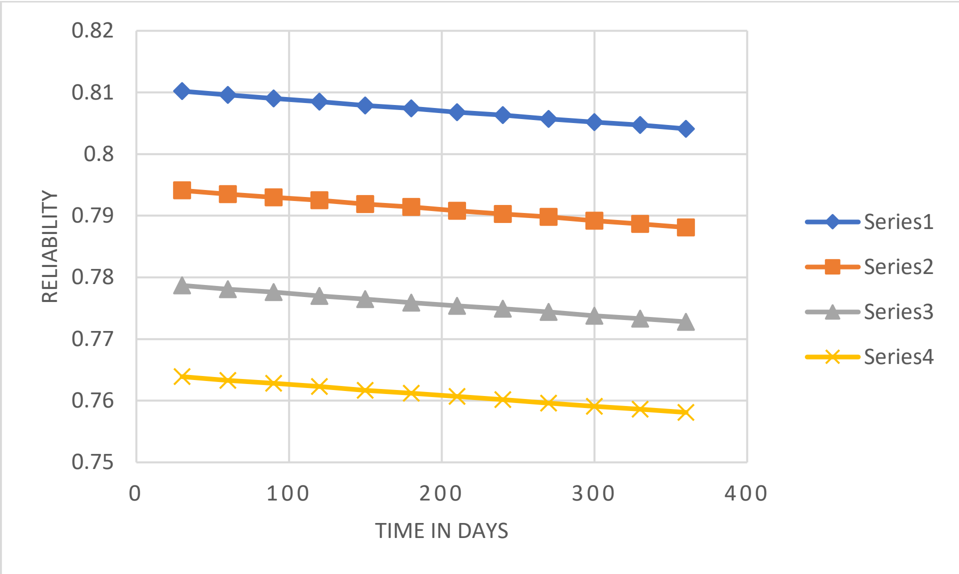
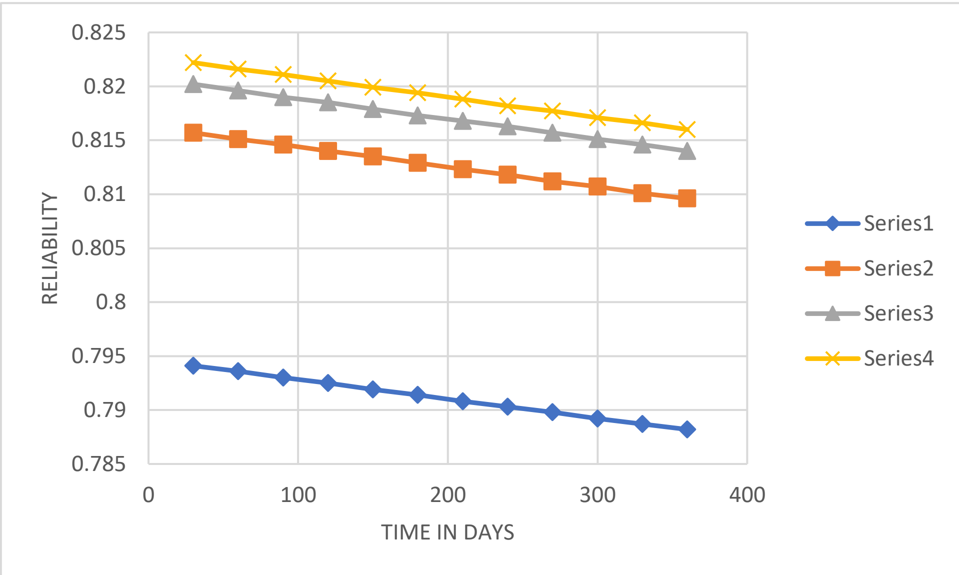


Figure-3



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