

Numerical Analysis of Topological Edge States in Photonic Crystals with Defect Engineering

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Abstract

Topological photonics has emerged as an important research area in optical physics because it enables electromagnetic waves to exhibit propagation characteristics associated with topological phases of matter. In particular, topological edge states can provide strongly confined and comparatively robust optical transport along interfaces between photonic structures with different topological properties. This paper presents a numerical framework for investigating topological edge states in two-dimensional photonic crystals and examines how deliberately engineered structural defects modify their spectral and spatial properties.

A two-dimensional dielectric photonic crystal is considered as the basic platform. The photonic band structure is obtained by solving the frequency-domain Maxwell eigenvalue problem using a plane-wave expansion or finite-element formulation. Topological characteristics are evaluated through Berry curvature and an appropriate topological invariant, while an interface between two photonic domains with different topological phases is introduced to generate localized edge states. Defects are subsequently introduced by modifying the radius, position, refractive index, or arrangement of selected dielectric elements near the interface. The resulting structures are analyzed in terms of band dispersion, localization length, transmission, field intensity, and sensitivity to structural perturbations.

The analysis predicts that the unperturbed interface supports modes inside the projected bulk band gap. Moderate defect engineering can shift the edge-state frequency, alter its localization length, and generate mode splitting or hybridization without necessarily destroying the bulk gap. Defects that preserve the relevant topological protection are expected to produce substantially smaller backscattering than conventional non-topological waveguide defects, whereas perturbations that break the protecting symmetry or couple counter-propagating states can significantly degrade robustness. Recent numerical and experimental studies support the use of defects as a means of tuning topological photonic states rather than treating defects solely as unwanted imperfections. The

proposed framework provides a basis for designing tunable topological waveguides, compact optical filters, sensors, and robust photonic integrated circuits.

Keywords

Topological photonics, photonic crystal, edge states, defect engineering, Berry curvature, Chern number, valley Hall effect, band gap, numerical simulation, optical waveguide.

1. Introduction

The manipulation and confinement of light at the nanoscale are fundamental objectives of modern photonics. Conventional optical waveguides guide electromagnetic energy through refractive-index contrast, while photonic crystals control light through periodic modulation of the dielectric function. Photonic crystals can exhibit photonic band gaps in which propagation is forbidden for selected frequency ranges. Introducing defects into these periodic structures can create localized optical states inside the band gap.

A fundamentally different approach to controlling light is provided by **topological photonics**. Topological photonic systems borrow concepts from condensed-matter physics and apply them to electromagnetic waves. The central idea is that global properties of the photonic bands can distinguish different phases even when their local material parameters appear similar. Interfaces between distinct topological phases can then support states that do not exist in either bulk system independently. The general development of this field and its implementation in photonic crystals are reviewed extensively in the literature.

The existence of an interface state is commonly associated with the principle of bulk-edge correspondence. In its simplest form, when two bulk photonic systems have different topological invariants, an interface between them must support states connecting the corresponding bulk bands.

Topological edge states are attractive because their propagation can be comparatively insensitive to certain types of disorder and imperfections. Early work demonstrated the possibility of topologically protected one-way photonic waveguides, while subsequent studies extended the concept to two-dimensional photonic crystals, valley photonics, higher-order topological systems, and topological defects.

However, the statement that topological modes are "immune to defects" requires qualification. Modern analyses emphasize that topological protection is not absolute. Perturbations that respect the relevant protecting conditions generally have limited impact, whereas perturbations that close the band gap, break the protecting symmetry, or couple otherwise separated states can substantially modify the modes.

This motivates the concept of **defect engineering**. Instead of treating defects merely as fabrication errors, defects can be deliberately introduced to manipulate topological optical states.

Recent research has demonstrated that structural defects can tune topological corner states, modify eigen frequencies and quality factors, and even transform optically dark states into bright states. Similarly, topological lattice defects have been used to generate protected photonic edge states associated with real-space topology.

The objective of this paper is therefore to develop a numerical framework for studying the relationship between:

photonic band topology → edge states → engineered defects

The study focuses on how controlled perturbations affect the frequency, localization, transmission, and robustness of topological edge states.

2. Background Theory

2.1 Photonic Crystals

A photonic crystal is a periodic dielectric structure satisfying

$\varepsilon(r + R) = \varepsilon(r)$, where R is any lattice translation vector.

For a two-dimensional photonic crystal,

$\varepsilon(x, y) = \varepsilon(x + Rx, y + Ry)$.

The periodic dielectric modulation causes electromagnetic waves to undergo Bragg scattering, resulting in allowed and forbidden frequency regions.

The forbidden region is called a **photonic band gap (PBG)**.

2.2 Maxwell Eigenvalue Equation

For a source-free, nonmagnetic medium, Maxwell's equations are

$$\nabla \times E = -\mu_0 \partial H / \partial t,$$

$$\nabla \times H = \varepsilon_0 \varepsilon_r(r) \partial E / \partial t.$$

Assuming harmonic time dependence,

$$E(r, t) = E(r)e^{-i\omega t},$$

the magnetic-field eigenvalue equation can be written as

$$\nabla \times [1\varepsilon_r(r)\nabla \times H] = \omega^2 c_2 H.$$

This equation forms the basis for numerical calculation of the photonic bands.

3. Bloch Theorem

Because the dielectric function is periodic, the electromagnetic field satisfies Bloch's theorem:

$$Hk(r) = eik \cdot ru k(r),$$

where

$$uk(r + R) = uk(r).$$

Here,

- k is the Bloch wave-vector;
- uk is a periodic function;
- $\omega(k)$ defines the photonic band structure.

The eigenvalue problem is solved over the first Brillouin zone.

For a two-dimensional hexagonal lattice, a typical high-symmetry path is

$$\Gamma \rightarrow M \rightarrow K \rightarrow \Gamma.$$

The resulting plot

$$\omega(k)$$

reveals the photonic bands and band gaps.

4. Topological Photonic Phases

Ordinary band structures do not contain sufficient information to distinguish all photonic phases. Topological characterization requires quantities constructed from the eigenstates.

For a normalized Bloch mode

$$|un(k)\rangle,$$

the Berry connection is

$$A_n(k) = i\langle un(k) | \nabla k$$

The Berry curvature is

$$\Omega_n(k) = \nabla k \times A_n(k).$$

In a two-dimensional system, the Chern number is

$$C_n = 12\pi \int_{BZ} \Omega_n(k) d^2k.$$

A nonzero difference in the relevant bulk topological invariants between two domains indicates the possibility of interface states.

5. Choice of Photonic Crystal

For the proposed numerical study, a two-dimensional dielectric photonic crystal consisting of high-index dielectric rods embedded in a lower-index background is considered.

The dielectric function is

$$\varepsilon(r) = \{\varepsilon_d, r \in \text{dielectric rods}, \varepsilon_b, r \in \text{background}\}.$$

A hexagonal or honeycomb lattice is particularly attractive because symmetry breaking can be used to create topologically distinct domains.

The lattice constant is denoted by

a .

The dielectric-rod radius is

r .

The refractive indices are

$$n_d = \varepsilon_d, n$$

and

$$n_b = \varepsilon_b, n$$

For numerical simulations, representative values can be selected according to the intended operating wavelength and fabrication platform.

6. Formation of Two Topologically Distinct Domains

A central component of the proposed model is an interface between two photonic domains.

Let the two domains be denoted by

A

and

B .

A structural parameter m is introduced to break the symmetry of the unit cell:

$$m_A = +m_0, m_{-A} = +m_0, m_B = -m_0, m_{-B} = -m_0.$$

This reverses the sign of the effective mass term near the relevant band degeneracy. The two domains therefore possess different topological characteristics.

An interface is then constructed as

Domain A | Domain B \boxed{\text{Domain A} \setminus \setminus \text{Domain B} }

At this boundary, an edge state can emerge inside the projected bulk band gap.

7. Topological Edge States

An edge state is an electromagnetic mode localized near the interface between two distinct photonic phases.

Its field can be approximately represented as

$$E(x, y) \sim e^{-\kappa |x|} e^{iky},$$

where

$$\kappa > 0 \setminus \kappa > 0$$

is the transverse decay constant.

The mode therefore propagates along the interface in the yy -direction while remaining confined in the xx -direction.

The dispersion relation takes the form

$$\omega = \omega(ky).$$

A topological edge state can often be identified numerically by a branch connecting bulk bands across the band gap.

8. Numerical Super cell Model

To calculate edge states, a super cell is constructed containing both topological domains.

The geometry can be written schematically as

$$\underbrace{A \setminus A \setminus A \setminus A}_{\text{Domain A}} | \underbrace{B \setminus B \setminus B \setminus B}_{\text{Domain B}} \setminus \setminus$$

Periodic boundary conditions are applied along the interface direction.

The super cell is sufficiently wide in the transverse direction that the two interfaces do not strongly interact.

The eigenvalue equation is then solved for a series of wave vectors

$$ky_k$$

along the interface.

The calculated frequencies are

$\omega_n(ky)$.

Plotting these frequencies produces the projected band structure.

Edge modes appear inside the bulk band gap.

9. Defect Engineering

The central purpose of this research is to investigate deliberately introduced defects.

A defect may be generated by changing:

- dielectric-rod radius;
- dielectric constant;
- rod position;
- lattice spacing;
- interface termination;
- missing rods;
- additional rods;
- local refractive index;
- interface geometry.

A general defect parameter can be defined as

$\delta = p - p_0$,

where p_0 is the unperturbed structural parameter and p is its perturbed value.

For example, if the defect is created by changing rod radius,

$\delta r = r_d - r_0$.

The numerical study can then investigate

$\delta r = -0.2, -0.1, 0, 0.1, 0.2$.

10. Types of Engineered Defects

10.1 Radius Defect

The radius of one or more dielectric rods near the interface is modified:

$r \rightarrow r + \Delta r$.

This changes the local dielectric filling fraction and consequently modifies the local optical potential.

10.2 Positional Defect

A dielectric element is displaced:

$$\mathbf{r}_i \rightarrow \mathbf{r}_i + \Delta \mathbf{r}.$$

This introduces controlled structural disorder.

10.3 Missing-Element Defect

One dielectric rod is removed from the lattice.

This is particularly useful for investigating the ability of the topological edge state to propagate past a strong local perturbation.

10.4 Interface Defect

The interface itself can be modified by introducing:

- bends;
- notches;
- steps;
- localized changes in lattice geometry.

This permits investigation of edge-state transmission around geometrical imperfections.

11. Numerical Methodology

The numerical analysis can be performed using one of the following methods:

1. Plane Wave Expansion (PWE);
2. Finite Element Method (FEM);
3. Finite Difference Frequency Domain (FDFD);
4. Finite Difference Time Domain (FDTD).

For the present study, the **finite-element eigenvalue method** is particularly suitable because it permits accurate treatment of complicated defect geometries.

Commercial software such as COMSOL Multiphysics or an open numerical implementation can be used.

The general workflow is:

Geometry → Mesh → Maxwell eigenproblem → Band structure → Topology → Defect analysis

12. Band-Structure Calculation

For each Bloch wavevector $\mathbf{k}$, solve

$$\nabla \times [1/\epsilon_r \nabla \times \mathbf{H}] = \omega^2 \epsilon_0 \epsilon_r \mathbf{H}.$$

The resulting eigenfrequencies are

$$\omega_1(\mathbf{k}), \omega_2(\mathbf{k}), \dots$$

The normalized frequency is defined as

$$\Omega = \omega a / 2\pi c = f a / c.$$

The photonic band gap is identified as

$$\Omega_{\text{lower}} < \Omega < \Omega_{\text{upper}}.$$

13. Identification of Edge Modes

For the interface super cell, the projected bulk bands are calculated.

An edge state is identified when:

1. its frequency lies within the projected bulk band gap;
2. its electromagnetic energy is strongly localized near the interface;
3. its dispersion connects the appropriate bulk bands.

A localization measure can be defined as

$$\eta = \frac{\int_{\text{interface}} |E|^2 dA}{\int_{\text{entire supercell}} |E|^2 dA}.$$

A large value of η indicates strong edge localization.

14. Localization Length

The field intensity perpendicular to the interface can be approximated by

$$I(x) = I_0 e^{-2\kappa|x|}, I(x) = I_0 e^{-2\kappa|x|}.$$

The localization length is

$$\xi = 1/2\kappa.$$

Alternatively, ξ can be obtained by fitting the numerically calculated field profile to an exponential function.

A smaller ξ corresponds to stronger confinement around the interface.

15. Defect-Induced Frequency Shift

Suppose the edge-state frequency without a defect is

$$\omega_0.$$

After introducing a defect, it becomes

$$\omega_d.$$

The normalized frequency shift is

$$\Delta\Omega = \omega_d - \omega_0.$$

This quantity provides a convenient measure of the sensitivity of the topological state to defect engineering.

For small perturbations,

$$|\Delta\Omega| \ll 1$$

would indicate relatively weak spectral sensitivity.

16. Transmission Analysis

To investigate the robustness of the edge state, a finite-length waveguide is constructed.

An excitation is launched at one end of the interface.

The transmitted power is

$$P_{\text{out}} = \int \mathbf{S} \cdot d\mathbf{A},$$

where the Poynting vector is

$$\mathbf{S} = \frac{1}{2} \text{Re}(\mathbf{E} \times \mathbf{H}^*).$$

The transmission coefficient is

$$T = P_{\text{out}} / P_{\text{in}}.$$

For a robust edge mode, one expects high transmission over the relevant frequency interval even when moderate defects are introduced.

17. Backscattering Analysis

A useful measure of defect sensitivity is the reflected power

$$R = P_{\text{back}} / P_{\text{in}}.$$

The scattering loss can be estimated through

$$L_s = -10 \log_{10} T, L_s = -10 \log_{10} T.$$

Comparison between topological and conventional waveguides provides a meaningful measure of the benefit of topological design.

For example,

$$\Delta L_s = L_{\text{conventional}} - L_{\text{topological}}.$$

A positive value indicates lower scattering loss for the topological configuration.

However, topological protection should not be interpreted as complete immunity. Modern reviews emphasize that protection depends on the particular topology, symmetry, disorder, and coupling channels of the system.

18. Defect-Engineering Parameter Study

The following numerical parameter sweep is proposed.

Parameter	Values to investigate
Defect radius variation $\Delta r/r$	-20%-20\% to +20%+20\%
Positional displacement $\Delta x/a$	00-0.250.25
Defect refractive index	Several values
Number of defects	1-5
Interface bend angle	0°0^\circ-90°90^\circ
Defect location	Core/interface/background
Operating frequency	Across topological band gap

For each configuration, calculate:

$$\omega_d, \xi, T, R, Q.$$

19. Expected Numerical Results

The unperturbed photonic crystal should first exhibit a bulk band gap.

After creating the interface between the two topologically distinct domains, an edge-state branch should appear inside the projected band gap.

The edge-state branch should be spatially localized at the interface.

When a moderate defect is introduced, the following behaviour is expected:

$$\omega_0 \rightarrow \omega_d \backslash \omega_{a_0}$$

with a relatively small modification of the propagating edge branch when the defect preserves the relevant topological protection.

The field may locally deform around the defect but continue along the interface.

This general behaviour is consistent with the broader literature on topological photonics, where interfaces between topologically distinct phases support robust guided states.

20. Strong Defects and Breakdown of Protection

The response becomes more complicated for strong perturbations.

If a defect is sufficiently large to close the local band gap,

$E_g \rightarrow 0$,

the topological interface state may cease to exist.

Similarly, if the defect breaks the symmetry responsible for a particular topological phase, the protection mechanism can be weakened or eliminated.

Therefore, the expected relationship is not simply

larger defect $\Rightarrow$ no effect.

Rather,

weak symmetry-preserving defect $\rightarrow$ small perturbation

while

strong or symmetry-breaking defect $\rightarrow$ significant modification

This distinction is important in interpreting numerical simulations.

21. Defect-Induced Mode Splitting

When two topological states are brought into close proximity through defect engineering, they can hybridize.

Suppose two uncoupled modes have frequencies

ω_1, ω_2 .

Their coupling can produce

$$\omega_{\pm} = \omega_1 + \omega_2 \pm \sqrt{(\omega_1 - \omega_2)^2 + 4|\kappa|^2},$$

where κ is an effective coupling coefficient.

Thus, defect engineering can be used deliberately to control mode splitting rather than simply to test robustness.

Recent work has demonstrated that strategically introduced defects can tune eigenfrequencies, quality factors, and band structures of coupled topological corner states.

22. Quality Factor

For a resonant defect state, the quality factor is

$$Q = \omega_0 / \Delta\omega,$$

where $\Delta\omega$ is the resonance line width.

Alternatively, for a lossy numerical eigenmode,

$$Q = \text{Re}(\omega) / 2|\text{Im}(\omega)|.$$

Defect engineering can therefore be used to optimize

Q .

while maintaining topological localization.

This creates a potential route toward compact high- Q optical resonators and topological lasers. Recent reviews identify topological photonic crystals and related structures as promising platforms for compact lasers, robust optical states, and other integrated photonic devices.

23. Comparison with Conventional Defect Waveguides

A conventional photonic-crystal defect waveguide relies on a localized defect state inside a band gap. A topological waveguide instead uses an interface between distinct topological phases.

The difference can be summarized as follows:

Property	Conventional defect mode	Topological edge mode
Origin	Local structural defect	Difference in bulk topology
Localization	Usually defect-centered	Interface-centered
Disorder tolerance	Generally limited	Can be enhanced for suitable perturbations
Backscattering	Can be significant	Can be suppressed under appropriate protection
Band-gap requirement	Usually required	Bulk gap generally required
Defect engineering	Changes local resonance	Can tune interface state
Robustness	Geometry-dependent	Topology- and symmetry-dependent

Topological protection is therefore best understood as an additional design mechanism rather than a universal immunity to imperfections.

24. Proposed Results Table

The final numerical study can be reported using a table such as:

Defect strength	Edge-state frequency	Localization length	Transmission	Reflection
0%	ω_0	ξ_0	T_0	R_0
5%	To calculate	To calculate	To calculate	To calculate
10%	To calculate	To calculate	To calculate	To calculate
15%	To calculate	To calculate	To calculate	To calculate
20%	To calculate	To calculate	To calculate	To calculate

These values should be obtained from the numerical eigenmode and scattering simulations rather than inserted as assumed results.

25. Numerical Implementation Procedure

The complete simulation procedure can be summarized as follows.

Phase I — Bulk crystal

1. Define the primitive unit cell.
2. Assign dielectric constants.
3. Generate the computational mesh.
4. Apply Bloch boundary conditions.
5. Calculate eigenfrequencies.
6. Plot the band structure.
7. Identify the bulk band gap.

Phase II — Topological characterization

1. Calculate the Bloch eigenstates.
2. Calculate Berry connection.
3. Calculate Berry curvature.
4. Evaluate the appropriate topological invariant.
5. Confirm that the two domains belong to distinct phases.

Phase III — Interface

1. Construct a super cell containing domains A and B.
2. Apply periodicity along the interface.
3. Calculate the projected band structure.
4. Identify modes inside the bulk band gap.
5. Plot the electric/magnetic field distribution.

Phase IV — Defect engineering

1. Introduce one controlled defect.
2. Recalculate eigenfrequencies.
3. Determine frequency shift.
4. Calculate localization length.
5. Repeat for multiple defect strengths.

Phase V — Transmission

1. Construct a finite waveguide.
2. Excite the topological edge state.
3. Introduce defects along the propagation path.
4. Calculate transmitted and reflected power.
5. Compare with an equivalent conventional waveguide.

26. Discussion

The proposed model illustrates the central advantage of topological photonic structures: modal behavior can be determined not only by local refractive-index distributions but also by global properties of the photonic bands.

The edge state originates from the interface between two distinct topological phases. Consequently, a local perturbation does not necessarily remove the state.

However, the effect of a defect depends strongly on its nature.

A small change in the dielectric radius may primarily shift the local resonance. A positional perturbation may cause local distortion of the field. A missing dielectric element can introduce stronger scattering. An interface bend can test the ability of the edge mode to follow a non-straight propagation path.

This makes defect engineering a particularly useful numerical tool.

Rather than asking simply whether a topological state survives a defect, the more useful research question is:

How can defects be engineered to control the properties of topological states?

Possible quantities to optimize include

$$\omega, Q, \xi, T, R, \omega_0, Q, \xi, T, R.$$

This perspective is increasingly relevant because modern topological photonics is moving from demonstrations of robustness toward practical device engineering. Recent research includes topological corner states, real-space topological defects, topological lasers, and defect-mediated optical functionality.

27. Potential Applications

27.1 Robust Optical Interconnects

Topological edge states may enable optical signals to propagate along integrated photonic structures with reduced sensitivity to certain forms of disorder.

27.2 Optical Filters

Defect-induced shifts of edge-state frequencies can be exploited to produce tunable spectral filters.

27.3 Optical Sensors

Because local defects modify the electromagnetic field and resonance frequency, topological modes can be used for sensing environmental changes.

A 2026 study, for example, investigated topological light trapping in photonic-crystal biosensors, using domain-wall defect modes to concentrate optical fields in sensing regions.

27.4 Topological Lasers

Topological photonic structures can provide robust mode selection and engineered optical confinement. Recent reviews identify Chern, valley-Hall, higher-order, and related photonic structures as promising routes for compact laser architectures.

27.5 Integrated Photonic Circuits

Topological waveguides may be incorporated into compact circuits containing bends, junctions, resonators, and engineered scattering centers.

28. Future Scope

The study can be extended in several directions.

28.1 Non-Hermitian topological photonics

Gain and loss can be introduced through a complex dielectric function,

$$\varepsilon = \varepsilon' + i\varepsilon''.$$

This enables investigation of exceptional points and non-Hermitian topological transitions.

28.2 Machine-learning-assisted defect design

A machine-learning model can be trained to predict

$\{geometry\} \rightarrow \{\omega, Q, T, \xi\}$.

Optimization algorithms could then identify defect configurations producing a desired optical response.

28.3 Nonlinear topological photonics

A nonlinear refractive index can be included as

$$n = n_0 + n_2 |E|^2. n = n_0 + n_{-2} |E|^2.$$

This permits investigation of intensity-dependent topological modes.

28.4 Dynamic topological systems

Time-dependent modulation can be introduced:

$$\epsilon(r,t) = \epsilon_0 + \Delta\epsilon \cos(\Omega t).$$

This could produce dynamically controlled topological transitions.

28.5 Higher-order topology

Instead of one-dimensional edge states, the system can be engineered to support zero-dimensional corner states.

Topological corner states and defect engineering are already active research directions.

29. Conclusion

A numerical framework for investigating topological edge states in photonic crystals with engineered defects has been presented.

The study begins with a periodic dielectric photonic crystal and calculates its band structure using the frequency-domain Maxwell eigenvalue equation. Topological properties are then characterized using Berry curvature and an appropriate bulk invariant. Two photonic domains with distinct topological characteristics are joined to form an interface, producing localized edge states within the projected bulk band gap.

Controlled structural defects are subsequently introduced near the interface. Defects may involve variations in dielectric-rod radius, position, refractive index, lattice geometry, or interface structure. Their effects are quantified through changes in edge-state frequency, localization length, quality factor, transmission, and reflection.

The central conclusion is that defect engineering should not be viewed solely as a test of topological robustness. It can also serve as a deliberate design mechanism for controlling optical modes. Moderate perturbations that preserve the relevant topological protection can leave the edge state largely intact while producing useful spectral and spatial modifications. Strong perturbations or

symmetry-breaking defects can substantially alter the state and may eventually destroy the relevant topological protection.

Thus, the combination of photonic topology and controlled defect engineering provides a promising route toward tunable and robust optical components. Potential applications include integrated optical interconnects, filters, sensors, resonators, and topological lasers.

The proposed numerical methodology provides a reproducible foundation for future computational studies using FEM, PWE, FDTD, or related electromagnetic simulation techniques.

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