



Chaotic parameter control in Differential Evolution

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Abstract

Differential Evolution (*DE*) is a popular meta-heuristic algorithm that works on the differences between the vectors of a population. It was developed by Kenneth Price and Rainer Storn in the mid-1990s. The scaling factor F and the crossover probability CR are two key control parameters of *DE*. Differential Evolution (*DE*) works well on many optimization problems, but its results depend a great deal on how the scaling factor F and the crossover probability CR are set. In this paper, the concepts of dynamical systems (chaotic maps) and population diversity are applied to the control parameters F and CR . We replace the fixed values of these two parameters with sequences drawn from ten chaotic maps: Chebyshev, circle, Gauss, iterative, logistic, piecewise, sine, Singer, sinusoidal and tent. A diversity check on the population decides when the chaotic values are used; otherwise $F = 0.5$ and $CR = 0.4$ are kept. We tried each map on F alone, on CR alone, and then in pairs chosen for each class of problems. All variants were run on the 30 CEC 2014 functions and compared with standard *DE*. No single map won everywhere. These findings can help in choosing a chaotic map to suit the type of problem at hand.

Keywords

Differential evolution; Chaotic maps; Population diversity; Scaling factor; Crossover probability; CEC 2014

1. Introduction

Meta-heuristic algorithms have attracted considerable interest in recent times. The field is relatively new, and its applications arise naturally from the need to solve high-dimensional, non-linear problems that traditional optimization techniques cannot handle. Low computational cost, flexibility and ease of adaptation are the key reasons for the popularity of heuristic algorithms. Among them, Differential Evolution (*DE*), proposed by Kenneth Price and Rainer Storn, is a direct-search, population-based stochastic method. Other remarkable algorithms include Particle Swarm Optimization (PSO), Artificial Bee Colony (ABC), Gravitational Search Algorithm (GSA), Ant Colony Optimization (ACO), Biogeography-Based Optimization (BBO), Grey Wolf Optimizer (GWO), Spider Monkey Optimization (SMO) and many more. *DE* has become a major area of interest for researchers, and its simplicity and strong performance justify this attention. The two main characteristics of any population-based method are exploration and exploitation. Exploration is the ability to cover the whole search space, which begins with the random initialization of the vectors; large changes help exploration and increase the chance of finding the optimal region. To reach the solution, local or gradient-based search is then used to refine the optimum. This

is termed exploitation: small changes are made in the local or neighbourhood area. Both exploration and exploitation are equally important for an algorithm to be efficient and reliable. Algorithms usually suffer from an imbalance between the two, which causes premature convergence or stagnation before the optimum is reached. This raises the question of how the performance of an algorithm can be improved by means that are low in computational cost and easy to apply. DE has attracted both academicians and industrialists, and many improved methods, such as adaptive JADE [21] and self-adaptive DE [11], have been introduced. Many variants of DE have also been proposed, such as DE with auto-enhanced population diversity [20]. Moreover, DE has been applied to IIR filter design [18], to self-potential data [6], and to job-shop scheduling through a hybrid DE-tabu search [14]. Research on Differential Evolution (DE) and chaos-based search techniques has continued to attract considerable attention in recent years. Chakraborty et al. [2] presented a comprehensive review of DE research over previous twelve years. In contrast, Reyes-Davila et al. [16] focused specifically on DE operators. Pluháček et al. [13] conducted a bibliographic analysis of chaos-enhanced metaheuristics. Türkoğlu et al. [19] discussed the potential of chaotic maps to regulate the balance between exploration and exploitation. Similarly, Lozi [7] reviewed recent applications of the Lozi map, including its integration into DE and Particle Swarm Optimization (PSO). Focusing on practical optimization problems, Parra-Ocampo et al. [12] developed sixteen chaotic DE variants by incorporating logistic, sine, Hénon, and Lozi sequences into different algorithmic stages, namely initialization, mutation, and crossover. To address this limitation, the present study systematically evaluates ten chaotic maps on all 30 benchmark functions of the CEC 2014 test suite. This approach enables a comparative investigation of the effectiveness of different chaotic maps and their influence on the search behaviour of DE.

This paper compares the performance of different chaotic maps in DE. Rather than using random sequences, sequences generated by chaotic maps are applied to the two control parameters of DE. The low computational cost of chaotic maps also encourages their use with DE. The nature of the chaotic maps is discussed in detail in Section 4. The rest of the paper is organized as follows. Section 2 gives an overview of basic DE. Section 3 reviews related modifications of DE and work on chaos theory. Chaotic maps and diversity are introduced in Section 4. The experimental results are discussed in Section 5, and conclusions are drawn in Section 6.

2. Brief Overview of DE

Differential Evolution belongs to the class of evolutionary algorithms. It is a population-based technique that is stochastic in nature. DE differs from several other evolutionary algorithms in that its mutation operator is defined in terms of scaled difference vectors and is applied before the crossover operator. The three main operators of DE are mutation, crossover and selection. Price, Storn and Lampinen [15] described DE as a fast and reliable algorithm because it has few control variables: the scaling factor (F), the crossover probability (CR) and the population size (NP). NP is the number of candidate solutions in the search space, and choosing a suitable population size is equally important. It has been suggested [15] that NP should lie in the range $[5D, 10D]$, where D is the dimension of the test problem.

2.1 Process of DE

Initialization: The first step is to sample NP individuals from the search space. For a D-dimensional search space, each candidate is represented as a vector $(x_{i1}, x_{i2}, x_{i3}, \dots, x_{iD})$ $i=1,2,3,\dots,NP$. Each target vector is initialized using

$$X_i(G) = L_j + rand(0, 1) \times (U_j - L_j) \quad (1)$$

where L_j and U_j represents the Lower Bound and Upper Bound of the j^{th} variable, respectively

Mutation: The DE mutation operator generates a mutant vector for each individual of the current population. The mutant vector is generated using Eq. (2), where $x_{i_1}, x_{i_2}, x_{i_3}$ are 3 randomly selected vectors $r1 \neq r2 \neq r3$

$$v_i(G+1) = x_{i_1}(G) + F(x_{i_2}(G) - x_{i_3}(G)) \quad (2)$$

where $v_i(G+1)$ is the mutant vector and G is the generation index. The scaling factor F is generally advised to be taken from $[0, 2]$ [15]. In numerical experiments *DE* performs better in the range $(0.5, 1)$, and only rarely are extraordinary results obtained for values greater than 1.

There are many working strategies of *DE*. The most popular and basic strategy is *DE/rand/1*. Other strategies, such as *DE/best/1* and *DE/current-to-rand/1*, have been applied by researchers, but according to the No Free Lunch theorem no strategy can be claimed to be the best for all problems.

Crossover: After mutation, crossover generates the trial vector $u_i(G)$ by recombining the components of the mutant vector $v_i(G)$ with those of the target vector $x_i(G)$. It is performed to increase the diversity of the population. The crossover probability CR is the probability that a component is taken from the mutant vector; if CR is high, more components are inherited from the mutant vector. In this paper binomial crossover is used, so only this operator is discussed:

$$u_{i,j}(G) = \begin{cases} v_{i,j}(G), & \text{if } rand_j(0,1) < CR \text{ or } j = j_{rand} \\ x_{i,j}(G) & \text{otherwise} \end{cases} \quad (3)$$

In Eq. (3), the condition $j = j_{rand}$ guarantees that at least one component of the trial vector comes from the mutant vector; if neither condition holds, the component of the target vector is retained.

Selection The last operator is selection, which decides whether the trial vector or the target vector survives into the next generation. For a minimization problem, the candidate with the lower objective value survives, according to Eq. (4):

$$X_i^{(G+1)} = \begin{cases} U_i^{(G)}, & \text{if } f(U_i^{(G)}) < f(X_i^{(G)}) \\ X_i^{(G)} & \text{otherwise} \end{cases} \quad (4)$$

The pseudo code of DE is given below

3. Recent Works in Modification and Chaos

A great deal of work on DE has either hybridized DE with other population-based stochastic methods, introduced new control parameters, or fine-tuned the existing control parameters. Mallipeddi and Suganthan [8] proposed a modification in which each candidate is assigned a mutation strategy from a pool of strategies, and the candidate is re-initialized if better offspring are not found. Gong and Cai [4] introduced ranking-based mutation operators, inspired by BBO, in which a vector with a higher rank has a greater chance of being selected. The dynamic scaling factor approach [17] changes the step size dynamically from $[0.8, 0.8]$ to $[0.4, 0.4]$ to balance exploration and exploitation; the same authors also proposed a self-adaptive version with a new cognitive learning factor and dynamic values of F . Guo et al. [5] proposed self-adaptive differential evolution with global neighbourhood search (NSSDE), which modifies the scaling factor and crossover rate using a global neighbourhood search strategy to refine the quality of the candidate solutions. More recently, ANDE [10] introduced a triangular mutation rule based on a convex combination and applied it together with *DE/rand/1/bin* according to a probability. In this paper, different chaotic strategies are implemented in DE using chaotic maps together with diversity. The control parameters F and CR are given chaotic sequences according to the chaotic map applied. Chaotic

Algorithm 1 Differential Evolution:

```
Generate Population(G). Initialize the control parameters,  $F$  and  $CR$ .
for  $i=1$  to  $NP$  do
    for  $j=1$  to  $D$  do
         $X_i(G) = L_j + rand(0, 1) \times (U_j - L_j)$ .
    end for
    Evaluate the fitness for all  $X_i(G)$   $i = 1, 2 \dots NP$ 
end for
while end criterion is not satisfied. do
    for each individual  $i = 1, 2 \dots NP$  do
        Randomly select three distinct vectors r1, r2, r3 such that index of  $r1 \neq r2 \neq r3$  excluding  $x_i(G)$ .

        Create the mutant vector  $v_i(G)$  by applying Eq.(2) i.e. mutation operator.
        Create the trial vector  $u_i(G)$ , by applying Eq.(3) i.e. crossover operator.
        Selection operator is applied and fitness values are compared.
        if  $f(u_i(G))$  is better than  $f(x_i(G))$  then
            Add  $u_i(G)$  to  $(G + 1)$ 
        else
            Add  $x_i(G)$  to  $(G + 1)$ 
        end if
    end for
end while
```

maps arise from chaos theory. Chaos is a deterministic, random-like process found in non-linear dynamical systems; it is non-periodic, non-convergent and bounded [1]. Chaotic sequences are easily generated from an initial value given to the chaotic map (function). Owing to sensitivity to initial conditions, a slight variation in the initial value completely changes the generated sequence. The sequences are deterministic and can be reproduced, which is the quasi-stochastic property of chaotic maps. Chaotic sequences have been adopted in place of regular random sequences, which provide only stochasticity to the algorithm. In recent years, coupling meta-heuristic algorithms with dynamical systems has been shown to improve exploration and to help solutions escape local optima better than the original algorithms, thereby improving the solutions and their convergence. Chaotic maps have been used successfully in ABC by Alatas [1] to improve convergence. Alatas has also used different chaotic maps in PSO, and Gandomi et al. [3] proposed a chaotic accelerated PSO in 2013. Mirjalili and Gandomi [9] recently worked on a chaotic GSA that balances the trade-off between exploration and exploitation by applying ten different chaotic maps to the gravitational constant G . Inspired by the chaotic GSA, chaotic maps are adapted into DE, and a comparison is presented of which chaotic map works best for different kinds of test problems. The study covers four categories of problems in the 30 CEC 2014 benchmark functions [16]. The next section describes the chaotic maps and their properties and illustrates the use of diversity.

4. Chaos and Diversity

This section first describes the ten chaotic maps, as discussed in [9], and the way they are used to modify the algorithm. In the equations below, Ch_i is the value of the map at iteration i .

4.1 Chebyshev map

$$Ch_{i+1} = \cos(i.\arccos(Ch_i)). \quad (5)$$

The range of this map is $(-1, 1)$.

4.2 Circle map

$$Ch_{i+1} = \text{mod}(Ch_i + b - (\frac{a}{2\pi}) \sin(2\pi Ch_i), 1), a = 0.5, b = 0.2, \quad (6)$$

The range of this map is $(0, 1)$

4.3 Gauss Map

$$Ch_{i+1} = \begin{cases} 1 & Ch_i = 0 \\ \frac{1}{\text{mod}(Ch_i, 1)} & \text{otherwise} \end{cases} \quad (7)$$

4.4 Iterative map

$$Ch_{i+1} = \sin(\frac{a\pi}{Ch_i}), a = 0.7, \quad (8)$$

The range of maps above lies in the interval of $(-1, 1)$

4.5 Logistic map

$$Ch_{i+1} = aCh_i(1 - Ch_i), \text{ where } a=4 \quad (9)$$

the range of this map is $(0, 1)$

4.6 Piecewise Map

$$Ch_{i+1} = \begin{cases} \frac{Ch_i}{P} & 0 \leq Ch_i < P \\ \frac{Ch_i - P}{0.5 - P} & P \leq Ch_i < 0.5 \\ \frac{1 - P - Ch_i}{0.5 - P} & 0.5 \leq Ch_i < 1 - P \\ \frac{1 - Ch_i}{P} & 1 - P \leq Ch_i < 1 \end{cases}, P = 0.4 \quad (10)$$

4.7 Sine map

$$Ch_{i+1} = \frac{a}{4} \sin(\pi Ch_i), \text{ where } a=4 \quad (11)$$

the range of this map is $(0, 1)$

4.8 Singer map

$$Ch_{i+1} = \mu(7.86Ch_i - 23.31Ch_i^2 + 28.75Ch_i^3 - 13.302875Ch_i^4), \mu = 1.07, \quad (12)$$

the range of this map is $(0, 1)$

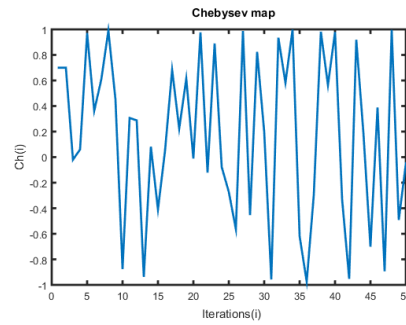


Figure 1

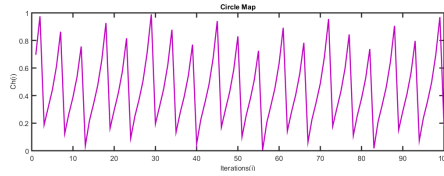


Figure 2

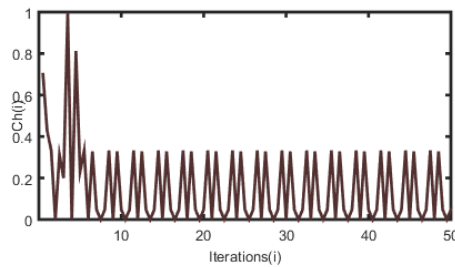


Figure 3

4.9 Sinusoidal Map

$$Ch_{i+1} = aCh_i^2 \sin(\pi Ch_i), a = 2.3 \quad (13)$$

the range of this map is (0,1)

4.10 Tent map

$$Ch_{i+1} = \begin{cases} \frac{Ch_i}{0.7}; & Ch_i < 0.7 \\ \frac{10}{3}(1 - Ch_i); & Ch_i \geq 0.7 \end{cases} \quad (14)$$

the range of this map is (0,1).

The pictorial view of the maps are shown figures 1-10.

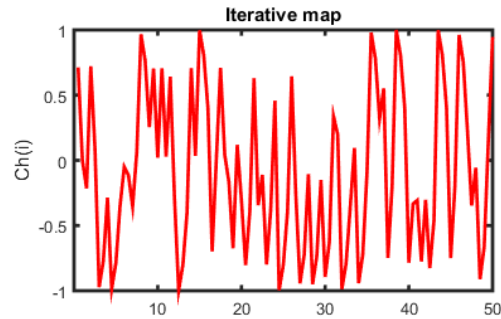


Figure 4

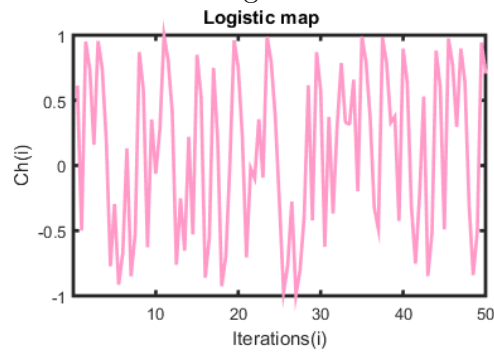


Figure 5

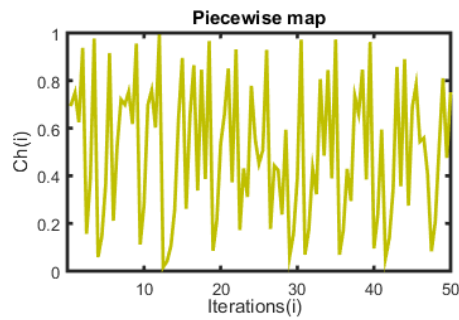


Figure 6

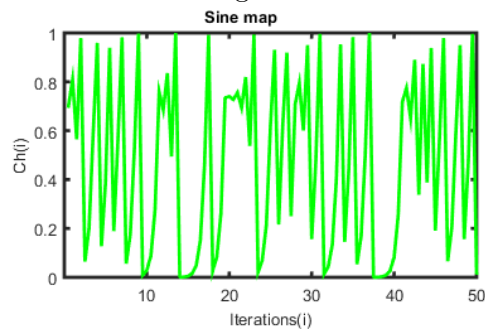


Figure 7

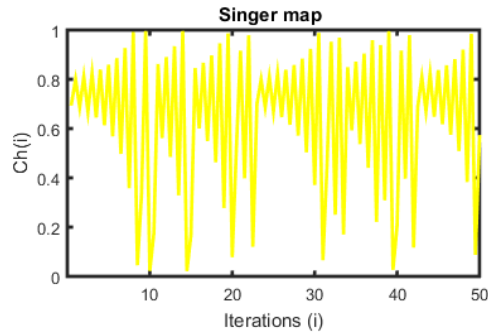


Figure 8

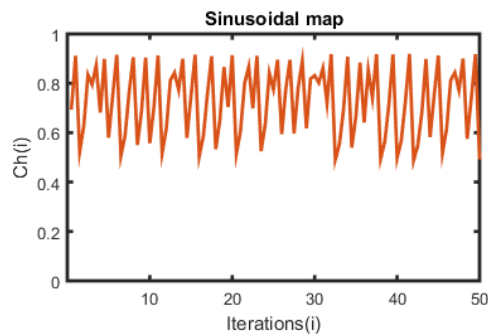


Figure 9

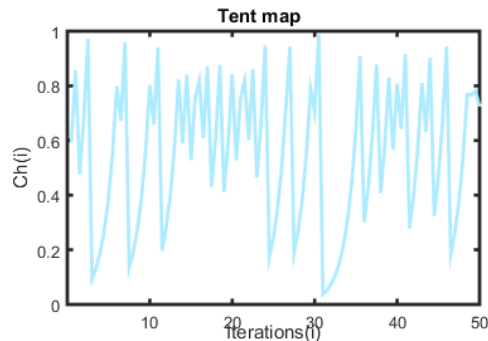


Figure 10

The control parameters F and CR are the main driving agents of the algorithm. They govern exploration and exploitation from the first to the last iteration. To enhance exploration, i.e. to move with large steps, the parameters take their values from the chaotic maps defined above. The aim is to provide the parameters with balanced changes rather than the abrupt changes of the random numbers used in the original DE . In simple terms, the nature of the different chaotic maps is utilized to enhance performance. Apart from the Chebyshev and iterative maps, all maps lie in the range $[0, 1]$; these two maps are normalized from $[1, 1]$ to $[0, 1]$. The initial value of every map is 0.7, except for the tent map, for which it is 0.6. Unlike the usual application of randomly generated numbers, chaotic maps generate the sequences in a prescribed fashion and not merely stochastically. Instead of applying the chaotic maps directly to the control parameters, diversity is used here as a catalyst.

Let LB_s and UB_s be the lower and upper bounds of the search space, and let $rand_j(0.5, 1)$ be a random number generated in the range $[0.5, 1]$. The chaotic values are used when

$$Diversity(G) < (UB_s - LB_s) \times rand_j(0.5, 1), \quad (15)$$

where Diversity(G) is the diversity of the population in generation G. The proposed modified algorithm is given below.

Algorithm 2 Modified Differential Evolution:

```
Generate Population(G).Initialize Diversity(G).
for i=1 to NP do
  for j=1 to D do
     $X_i(G) = L_j + rand(0, 1) \times (U_j - L_j)$ .
  end for
  Evaluate the fitness for all  $X_i(G)$   $i = 1, 2 \dots NP$ 
end for
while end criterion is not satisfied. do
  for each individual  $i = 1, 2 \dots NP$  do
    Calculate Diversity for  $G=G+1$ 
    if Eq. 15 is satisfied, use value of control parameters from Chaotic maps else take values as
    (constant) then
      Randomly select any three selected vectors r1,r2,r3 such that index of  $r1 \neq r2 \neq r3$  excluding
       $x_i(G)$ .
      Create the mutant vector  $v_i(G)$  by applying eq(2) ie mutation operator.
      Create the trial vector  $u_i(G)$ , by applying eq(3) ie crossover operator
      Selection operator is applied and fitness values are compared.
      if  $f(u_i(G))$  is better than  $f(x_i(G))$  then
        Add  $u_i(G)$  to  $(G+1)$ 
      else
        Add  $x_i(G)$  to  $(G+1)$ 
      end if
    end if
  end for
end while
```

In Algorithm 2, the values of F and CR are updated separately: first with the chaotic maps for the mutation operator only (F), and then separately for the crossover operator (CR). This gives a clear picture of which parameter benefits most from which chaotic map. The constant values are the standard settings $F = 0.5$ and $CR = 0.4$. The control parameters behave differently across problem classes: a map may work very well for one category and be less impressive for another. This gives readers and researchers an idea of how the approach can be applied in their own work.

5. Experimental Results and Discussion

5.1 Experimental setup

The modifications were tested on the CEC 2014 test suite. Functions $f1-f3$ are unimodal, $f4-f16$ are simple multimodal, $f17-f22$ are hybrid functions and $f23-f30$ are composition functions. Table 1 compares the mean error (MeanE), standard deviation (SD), median (MD), best and worst solutions of the original DE and of DE with the ten chaotic maps applied to $F(DE_F)$. Table 2 gives the same



statistics for the maps applied to $CR(DE_{CR})$. In this study the behaviour of the maps is analysed for all kinds of problems. No single best map is declared among all possibilities, whether the maps are applied to F , to CR or to both. The performance of a map is categorized on the basis of the modality and behaviour of the functions. The aim is not to emphasize improvements only, but to provide a broad classification and visualization of the maps in the algorithm; for example, a map that performs poorly in all considered cases can be disregarded by readers in future work, and vice versa.

Table 1: DE results with chaotic scaling factor $F(DE_F)$ on $f_1 - f_{30}$ (MeanE, SD , MD , Best, Worst)

Test problem	Algorithm	MeanE	SD	MD	Best	Worst
f_1	DE	5937.349	2772.828	5408.219	1707.588	12533.67
	$DE_F(Chebysev)$	13436.65	6712.107	14165.55	3898.445	30263.63
	$DE_F(Circle)$	3560.197	2296.761	3008.666	1185.885	10402.42
	$DE_F(Gauss)$	162320.2	543578.8	9262.345	11.37289	2181944
	$DE_F(Iterative)$	6617.774	3980.441	5668.727	2087.852	20742.92
	$DE_F(Logistic)$	4027.313	2513.775	3573.167	1413.325	9682.799
	$DE_F(Piecewise)$	1.49E-06	7.26E-06	1.01E-07	9.51E-09	4.01E-05
	$DE_F(Sine)$	2843.285	1376.804	2418.051	895.6047	5489.578
	$DE_F(Singer)$	15280.1	6158.987	15594.5	6353.847	23237.89
	$DE_F(Sinusoidal)$	29428.29	11721.48	27822.51	11068.77	70547.85
	$DE_F(Tent)$	5078.505	2859.983	4522.442	1743.48	10281.96
f_2	DE	8.12E-09	2.42E-09	8.41E-09	4.24E-09	9.98E-09
	$DE_F(Chebysev)$	8.12E-09	1.62E-09	8.28E-09	5.07E-09	9.90E-09
	$DE_F(Circle)$	8.21E-09	2.42E-09	8.22E-09	4.15E-09	9.97E-09
	$DE_F(Gauss)$	67031.67	208352.6	3297.913	0.74148	1140654
	$DE_F(Iterative)$	8.46E-09	1.90E-09	9.06E-09	2.47E-09	9.95E-09
	$DE_F(Logistic)$	8.09E-09	3.64E-09	8.16E-09	4.66E-09	9.90E-09
	$DE_F(Piecewise)$	7.74E-09	1.60E-09	7.81E-09	4.05E-09	9.89E-09
	$DE_F(Sine)$	8.51E-09	1.23E-09	8.68E-09	4.55E-09	9.99E-09
	$DE_F(Singer)$	7.93E-09	3.12E-09	8.26E-09	3.28E-09	9.95E-09
	$DE_F(Sinusoidal)$	8.21E-09	2.16E-09	8.55E-09	3.17E-09	1.10E-08
	$DE_F(Tent)$	8.16E-09	2.20E-09	8.74E-09	3.74E-09	9.96E-09
f_3	DE	7.85E-09	1.66E-09	8.11E-09	2.53E-09	9.79E-09
	$DE_F(Chebysev)$	8.21E-09	1.61E-09	8.84E-09	4.93E-09	9.96E-09
	$DE_F(Circle)$	8.39E-09	1.10E-09	8.62E-09	5.52E-09	9.95E-09
	$DE_F(Gauss)$	13.10343	27.58628	0.921548	9.66E-09	104.9595
	$DE_F(Iterative)$	7.64E-09	2.98E-09	8.14E-09	2.26E-09	9.96E-09
	$DE_F(Logistic)$	8.00E-09	1.41E-09	7.68E-09	5.03E-09	9.86E-09
	$DE_F(Piecewise)$	8.15E-09	1.50E-09	8.56E-09	4.25E-09	9.94E-09
	$DE_F(Sine)$	8.29E-09	1.47E-09	8.53E-09	4.13E-09	1.00E-08
	$DE_F(Singer)$	7.60E-09	1.88E-09	8.05E-09	2.58E-09	9.97E-09
	$DE_F(Sinusoidal)$	8.61E-09	1.48E-09	9.03E-09	4.54E-09	9.98E-09
	$DE_F(Tent)$	8.16E-09	1.80E-09	8.66E-09	4.43E-09	9.89E-09
f_4	DE	29.2802	13.0371	34.78027	0.437528	34.78027



f_5	$DE_F(Chebyshev)$	22.83592	19.76382	34.78027	0.251671	34.78027
	$DE_F(Circle)$	28.88975	13.59881	34.78027	0.420344	34.78027
	$DE_F(Gauss)$	25.29428	23.97224	34.78064	0.01721	35.52797
	$DE_F(Iterative)$	31.48437	32.25123	34.78027	0.336588	34.78027
	$DE_F(Logistic)$	28.65508	30.90869	34.78027	0.294854	34.78027
	$DE_F(Piecewise)$	24.68573	18.45394	34.78027	7.94E-09	34.78027
	$DE_F(Sine)$	21.39887	21.18798	34.78027	0.01213	34.78027
	$DE_F(Singer)$	23.15096	19.4438	34.78027	0.257783	34.78027
	$DE_F(Sinusoidal)$	22.95546	19.76295	34.78027	0.290505	34.78027
	$DE_F(Tent)$	28.05278	15.05874	34.78027	0.343129	34.78027
	DE	20.08192	1.447293	20.14811	18.6632	20.23561
f_6	$DE_F(Chebyshev)$	20.15233	0.043686	20.14935	20.07072	20.22805
	$DE_F(Circle)$	20.08515	0.460159	20.17105	17.61456	20.23481
	$DE_F(Gauss)$	20.10465	0.221258	20.1478	18.95613	20.21298
	$DE_F(Iterative)$	20.12544	0.218896	20.16555	19.00298	20.26807
	$DE_F(Logistic)$	20.15233	0.037878	20.14568	20.07803	20.22577
	$DE_F(Piecewise)$	20.14622	0.073461	20.15952	19.7924	20.22522
	$DE_F(Sine)$	20.14103	0.075277	20.1374	20.07524	20.21673
	$DE_F(Singer)$	20.1618	0.090353	20.18019	19.7166	20.25343
	$DE_F(Sinusoidal)$	20.17436	0.039878	20.17318	20.08041	20.24754
	$DE_F(Tent)$	20.15922	0.044896	20.15164	20.09721	20.24047
	DE	0.20218	0.437053	9.34E-09	4.35E-09	1.154703
f_7	$DE_F(Chebyshev)$	0.141094	0.348383	8.67E-09	1.94E-09	1.058049
	$DE_F(Circle)$	0.059651	0.23097	8.96E-09	5.09E-09	0.894543
	$DE_F(Gauss)$	0.190351	0.225127	0.080797	7.87E-06	0.993416
	$DE_F(Iterative)$	8.20E-09	1.49E-09	8.57E-09	4.77E-09	9.97E-09
	$DE_F(Logistic)$	0.03383	0.185294	8.07E-09	2.11E-09	1.014897
	$DE_F(Piecewise)$	0.113132	0.832737	8.72E-09	3.53E-09	1.058049
	$DE_F(Sine)$	0.102352	0.299963	8.81E-09	3.90E-09	1.014897
	$DE_F(Singer)$	0.029824	0.163321	8.99E-09	2.98E-09	0.894543
	$DE_F(Sinusoidal)$	0.089958	0.282908	9.68E-08	6.48E-09	0.894785
	$DE_F(Tent)$	0.097154	0.30915	9.16E-09	2.46E-09	1.125521
	DE	0.115159	0.182059	0.104702	0.025318	0.284757
f_8	$DE_F(Chebyshev)$	0.107274	0.065187	0.111362	3.50E-05	0.222827
	$DE_F(Circle)$	0.00468	0.008147	9.71E-09	4.52E-09	0.032128
	$DE_F(Gauss)$	0.035041	0.16827	0.017241	8.95E-09	0.342199
	$DE_F(Iterative)$	0.035576	0.09967	0.023673	7.10E-09	0.139518
	$DE_F(Logistic)$	0.009633	0.016033	0.007507	5.09E-09	0.062811
	$DE_F(Piecewise)$	0.032241	0.032153	0.024157	6.11E-09	0.099708
	$DE_F(Sine)$	0.007644	0.006564	0.008627	4.23E-09	0.024644
	$DE_F(Singer)$	0.072677	0.061137	0.059194	4.91E-09	0.191592
	$DE_F(Sinusoidal)$	0.2324	0.051334	0.228445	0.138146	0.3234
	$DE_F(Tent)$	0.017635	0.030792	0.005712	6.56E-09	0.096064
	DE	7.83E-09	1.84E-09	8.20E-09	4.44E-09	9.95E-09



f_9	$DE_F(Chebyshev)$	8.01E-09	2.47E-09	8.22E-09	4.82E-09	9.96E-09
	$DE_F(Circle)$	7.84E-09	2.54E-09	8.18E-09	3.74E-09	9.98E-09
	$DE_F(Gauss)$	1.268133	1.686708	0.99806	5.64E-09	3.979836
	$DE_F(Iterative)$	8.24E-09	3.41E-09	8.15E-09	5.06E-09	9.90E-09
	$DE_F(Logistic)$	0.033165	0.181654	8.85E-09	5.04E-09	0.994959
	$DE_F(Piecewise)$	7.77E-09	2.55E-09	8.28E-09	4.66E-09	9.99E-09
	$DE_F(Sine)$	0.099496	0.314634	8.39E-09	2.41E-09	0.994959
	$DE_F(Singer)$	0.033165	0.181654	8.70E-09	3.37E-09	0.994959
	$DE_F(Sinusoidal)$	8.20E-09	2.20E-09	8.65E-09	3.87E-09	9.99E-09
	$DE_F(Tent)$	0.033165	0.181654	8.16E-09	4.15E-09	0.994959
	DE	12.81808	2.240374	13.01878	8.56814	16.87345
f_{10}	$DE_F(Chebyshev)$	10.85529	4.001038	10.71828	6.928632	15.05243
	$DE_F(Circle)$	8.030059	3.560717	7.537939	4.991008	15.08518
	$DE_F(Gauss)$	4.497321	1.478882	4.216236	1.990159	7.988822
	$DE_F(Iterative)$	8.92633	2.375686	9.022244	3.750808	13.6165
	$DE_F(Logistic)$	2.631629	1.344934	1.996882	6.82E-09	5.268064
	$DE_F(Piecewise)$	8.087132	2.026826	7.965764	3.720676	12.01304
	$DE_F(Sine)$	2.327734	1.237464	1.989918	4.95E-09	4.974795
	$DE_F(Singer)$	11.4224	2.762238	11.4472	6.146924	15.61514
	$DE_F(Sinusoidal)$	15.45842	3.301505	14.99947	9.327378	20.83369
	$DE_F(Tent)$	7.634039	2.13538	7.479563	3.828066	11.77646
	DE	14.97587	14.3969	15.39293	0.249818	33.84444
f_{11}	$DE_F(Chebyshev)$	11.84209	9.590389	11.7983	8.32E-09	27.04519
	$DE_F(Circle)$	16.38942	10.11377	15.32375	0.187363	38.68735
	$DE_F(Gauss)$	23.62266	12.77384	23.26725	4.078123	58.50517
	$DE_F(Iterative)$	10.65627	13.04744	9.372716	0.124909	35.7331
	$DE_F(Logistic)$	15.78849	12.57584	15.11958	0.312272	35.85801
	$DE_F(Piecewise)$	12.76551	14.20367	14.26754	0.187363	27.38099
	$DE_F(Sine)$	17.48725	11.40744	13.87786	3.352506	48.55155
	$DE_F(Singer)$	13.13695	7.502999	12.13915	0.124909	27.31853
	$DE_F(Sinusoidal)$	18.08857	9.704174	18.62052	0.187363	35.33258
	$DE_F(Tent)$	16.59422	9.435778	15.29253	0.249818	31.76134
	DE	648.7775	163.5534	660.8578	368.1607	852.3998
f_{12}	$DE_F(Chebyshev)$	639.7887	122.831	637.0322	197.8424	789.2838
	$DE_F(Circle)$	582.5443	157.1485	563.5751	368.4735	806.2671
	$DE_F(Gauss)$	103.621	124.1682	51.70924	8.46518	502.199
	$DE_F(Iterative)$	564.6336	193.697	553.1231	141.2038	984.3905
	$DE_F(Logistic)$	475.7901	243.1672	507.6467	143.9792	802.8513
	$DE_F(Piecewise)$	560.4641	131.9481	565.8582	282.5584	770.8835
	$DE_F(Sine)$	390.9332	357.9768	420.9766	12.96348	768.996
	$DE_F(Singer)$	630.9541	156.6449	611.0518	316.3895	883.7292
	$DE_F(Sinusoidal)$	651.538	249.6073	663.6055	321.8816	990.8609
	$DE_F(Tent)$	571.5669	154.748	574.3395	217.9298	892.3482
	DE	0.49362	0.089748	0.488117	0.262867	0.656945



f_{13}	$DE_F(Chebyshev)$	0.502053	0.095527	0.50769	0.293293	0.715141
	$DE_F(Circle)$	0.513151	0.175046	0.527061	0.258926	0.684205
	$DE_F(Gauss)$	0.470468	0.11643	0.445894	0.259812	0.707852
	$DE_F(Iterative)$	0.459377	0.266168	0.477201	0.209779	0.716454
	$DE_F(Logistic)$	0.521307	0.106654	0.525549	0.304875	0.73509
	$DE_F(Piecewise)$	0.493855	0.13443	0.488392	0.249261	0.727543
	$DE_F(Sine)$	0.470677	0.106483	0.476858	0.214537	0.662425
	$DE_F(Singer)$	0.472659	0.108421	0.467143	0.25008	0.695503
	$DE_F(Sinusoidal)$	0.514026	0.100916	0.496173	0.322564	0.731228
	$DE_F(Tent)$	0.510766	0.154229	0.535084	0.267297	0.654696
	DE	0.16784	0.033808	0.172403	0.090462	0.218289
f_{14}	$DE_F(Chebyshev)$	0.168336	0.034804	0.169471	0.089418	0.230763
	$DE_F(Circle)$	0.145972	0.030121	0.153383	0.070656	0.204438
	$DE_F(Gauss)$	0.121529	0.044438	0.120127	0.080121	0.183679
	$DE_F(Iterative)$	0.149849	0.030852	0.142101	0.101093	0.211566
	$DE_F(Logistic)$	0.151445	0.024648	0.151749	0.110464	0.199861
	$DE_F(Piecewise)$	0.143629	0.023554	0.143916	0.098113	0.193552
	$DE_F(Sine)$	0.140318	0.034048	0.146103	0.06638	0.211137
	$DE_F(Singer)$	0.164813	0.032521	0.165448	0.111719	0.2316
	$DE_F(Sinusoidal)$	0.194923	0.078095	0.198669	0.113571	0.262538
	$DE_F(Tent)$	0.158886	0.0258	0.158938	0.105145	0.227803
	DE	0.159188	0.04596	0.162751	0.053444	0.234795
f_{15}	$DE_F(Chebyshev)$	0.168742	0.036231	0.164617	0.109151	0.250676
	$DE_F(Circle)$	0.181838	0.04517	0.178987	0.111933	0.271183
	$DE_F(Gauss)$	0.213466	0.083901	0.193989	0.086794	0.4293
	$DE_F(Iterative)$	0.174176	0.048911	0.171818	0.103732	0.259879
	$DE_F(Logistic)$	0.174979	0.049641	0.178218	0.097807	0.289407
	$DE_F(Piecewise)$	0.15103	0.036271	0.146336	0.080414	0.270478
	$DE_F(Sine)$	0.194757	0.061735	0.185535	0.105568	0.389896
	$DE_F(Singer)$	0.169872	0.065977	0.171848	0.098483	0.289882
	$DE_F(Sinusoidal)$	0.16007	0.055004	0.165612	0.065812	0.238497
	$DE_F(Tent)$	0.1578	0.045848	0.154856	0.102341	0.244746
	DE	1.456954	0.594387	1.489207	0.680875	2.059501
f_{16}	$DE_F(Chebyshev)$	1.627217	0.254914	1.670366	0.992671	2.039904
	$DE_F(Circle)$	1.397506	0.242332	1.44565	0.870316	1.898321
	$DE_F(Gauss)$	0.684311	0.193291	0.699072	0.366495	0.992546
	$DE_F(Iterative)$	1.554736	0.406161	1.586694	1.127685	2.077026
	$DE_F(Logistic)$	1.254668	0.22933	1.275036	0.716823	1.604311
	$DE_F(Piecewise)$	1.18144	0.279672	1.198386	0.795046	1.505504
	$DE_F(Sine)$	1.192846	0.166258	1.18286	0.806927	1.540685
	$DE_F(Singer)$	1.495652	0.297427	1.51853	0.878164	2.07419
	$DE_F(Sinusoidal)$	1.807616	0.848552	1.806816	1.021065	2.458726
	$DE_F(Tent)$	1.465138	0.233229	1.47573	1.082902	1.895623
	DE	2.564722	0.29272	2.630437	1.987858	2.943722



f_{17}	$DE_F(Chebyshev)$	2.549039	0.295057	2.559889	2.21149	2.864085
	$DE_F(Circle)$	2.456659	0.586272	2.485086	1.736731	2.954398
	$DE_F(Gauss)$	0.753498	0.390928	0.72267	0.252997	1.377193
	$DE_F(Iterative)$	2.513397	0.230794	2.523849	1.929778	3.057312
	$DE_F(Logistic)$	2.371109	0.390352	2.40501	1.852671	2.83584
	$DE_F(Piecewise)$	2.406829	0.307412	2.396707	1.742242	2.896944
	$DE_F(Sine)$	2.145839	0.53618	2.148255	1.128689	3.067422
	$DE_F(Singer)$	2.53465	0.275248	2.525263	2.006467	2.938875
	$DE_F(Sinusoidal)$	2.714547	0.205576	2.709759	2.218251	3.014118
	$DE_F(Tent)$	2.549564	0.355766	2.57839	1.949749	3.007089
	DE	58.34947	29.75083	58.51735	31.36662	102.0071
f_{18}	$DE_F(Chebyshev)$	68.15649	18.38814	67.19274	28.55461	106.6
	$DE_F(Circle)$	34.69314	24.04946	33.48562	9.070745	71.38493
	$DE_F(Gauss)$	184.4734	346.0922	29.28283	0.000121	1280.354
	$DE_F(Iterative)$	52.12233	27.62744	46.87236	27.64885	94.98732
	$DE_F(Logistic)$	25.02284	14.98703	20.11749	1.772621	57.80018
	$DE_F(Piecewise)$	9.909795	28.92356	2.132962	5.40E-07	150.7528
	$DE_F(Sine)$	16.50932	21.79233	15.33727	1.368172	38.3612
	$DE_F(Singer)$	73.51774	34.167	77.76973	23.66937	123.4766
	$DE_F(Sinusoidal)$	103.3453	99.92135	102.3573	48.89882	202.5663
	$DE_F(Tent)$	48.74736	40.16165	39.74779	20.96313	151.4087
	DE	1.712898	0.607489	1.707726	0.770484	3.006057
f_{19}	$DE_F(Chebyshev)$	1.87195	0.660099	1.946571	0.800404	3.335902
	$DE_F(Circle)$	1.186222	1.257431	1.25043	0.452565	2.31236
	$DE_F(Gauss)$	1846.994	3602.155	88.78549	1.38187	11743.26
	$DE_F(Iterative)$	1.590105	0.953756	1.718112	0.473904	2.632611
	$DE_F(Logistic)$	1.052867	0.870453	1.171567	0.087019	2.576202
	$DE_F(Piecewise)$	0.350327	0.335432	0.312812	0.021553	1.470904
	$DE_F(Sine)$	0.978824	0.669872	1.092098	0.052377	2.281448
	$DE_F(Singer)$	2.113962	0.586275	2.188409	0.85425	3.208526
	$DE_F(Sinusoidal)$	3.210727	1.065354	3.305228	1.569188	5.594508
	$DE_F(Tent)$	1.319548	0.792106	1.30026	0.282037	2.86607
	DE	0.270101	0.20881	0.247478	0.072506	0.584808
f_{20}	$DE_F(Chebyshev)$	0.285181	0.181597	0.267861	0.09359	0.670824
	$DE_F(Circle)$	0.164159	0.170783	0.142562	7.75E-05	0.492782
	$DE_F(Gauss)$	0.761553	0.563319	0.799908	0.093945	1.656167
	$DE_F(Iterative)$	0.263901	0.279651	0.207145	0.035271	0.692896
	$DE_F(Logistic)$	0.185016	0.876614	0.083004	0.019432	1.043152
	$DE_F(Piecewise)$	0.26211	0.312086	0.156934	0.078685	0.763621
	$DE_F(Sine)$	0.158572	0.272428	0.084164	0.019432	1.001614
	$DE_F(Singer)$	0.289497	0.113175	0.278907	0.134318	0.53454
	$DE_F(Sinusoidal)$	0.489787	0.327723	0.471084	0.210112	0.959606
	$DE_F(Tent)$	0.242401	0.245336	0.183681	0.022572	1.031291
	DE	0.117829	0.142417	0.092674	0.001166	0.476347



	$DE_F(Chebyshev)$	0.076474	0.095198	0.044773	0.002467	0.395297
	$DE_F(Circle)$	0.062886	0.0935	0.025158	0.000217	0.395328
	$DE_F(Gauss)$	44.62291	158.3152	1.08311	0.069856	756.8974
	$DE_F(Iterative)$	0.046386	0.083223	0.009248	6.27E-05	0.241421
	$DE_F(Logistic)$	0.11643	0.915062	0.034018	0.000164	1.026745
	$DE_F(Piecewise)$	0.036531	0.068422	0.00623	2.86E-05	0.299425
	$DE_F(Sine)$	0.107608	0.223403	0.037756	3.50E-06	1.097415
	$DE_F(Singer)$	0.058139	0.064175	0.035286	0.000153	0.219777
	$DE_F(Sinusoidal)$	0.388669	0.232367	0.318261	0.104274	1.149995
	$DE_F(Tent)$	0.044343	0.076547	0.011521	6.30E-06	0.339084
	DE	0.312439	0.284015	0.32769	0.047756	0.677014
f_{21}	$DE_F(Chebyshev)$	0.447767	0.4029	0.442282	0.065912	1.005287
	$DE_F(Circle)$	0.192791	0.241969	0.108442	0.01184	0.822521
	$DE_F(Gauss)$	22.18801	51.75854	4.502318	0.001378	157.2468
	$DE_F(Iterative)$	0.358717	0.579978	0.349769	0.038743	0.881363
	$DE_F(Logistic)$	1.400883	6.022351	0.347016	0.004272	33.55022
	$DE_F(Piecewise)$	0.338697	0.529924	0.279358	0.002019	1.123227
	$DE_F(Sine)$	0.316405	0.431503	0.31649	0.002207	1.056466
	$DE_F(Singer)$	0.396018	0.237102	0.328126	0.086399	1.188925
	$DE_F(Sinusoidal)$	1.168827	0.481836	1.141807	0.259554	2.52756
	$DE_F(Tent)$	0.342277	0.392631	0.38999	0.04345	0.691567
	DE	0.003328	0.00547	0.002121	9.29E-05	0.023571
f_{22}	$DE_F(Chebyshev)$	0.026392	0.078938	0.003633	0.000458	0.313056
	$DE_F(Circle)$	0.03024	0.078513	0.006293	4.43E-05	0.325856
	$DE_F(Gauss)$	3.170775	7.378329	0.432255	0.002388	20.86772
	$DE_F(Iterative)$	0.077965	0.151323	0.002872	2.24E-05	0.320741
	$DE_F(Logistic)$	0.07596	0.167482	0.013052	8.37E-07	0.626544
	$DE_F(Piecewise)$	0.761636	3.698442	0.004788	6.17E-05	20.23297
	$DE_F(Sine)$	0.111305	0.184363	0.021125	1.61E-06	0.631449
	$DE_F(Singer)$	0.015563	0.056644	0.004378	0.001305	0.312633
	$DE_F(Sinusoidal)$	0.12322	0.314261	0.040034	0.004834	1.661017
	$DE_F(Tent)$	0.067531	0.141274	0.00176	3.62E-07	0.322384
	DE	329.4575	0	329.4575	329.4575	329.4575
f_{23}	$DE_F(Chebyshev)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_F(Circle)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_F(Gauss)$	329.4687	0.056343	329.4576	329.4575	329.7661
	$DE_F(Iterative)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_F(Logistic)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_F(Piecewise)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_F(Sine)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_F(Singer)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_F(Sinusoidal)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_F(Tent)$	329.4575	0	329.4575	329.4575	329.4575
	DE	329.4575	0	329.4575	329.4575	329.4575



f_{24}	DE	119.9061	3.33197	119.7548	115.6218	127.7835
	$DE_F(Chebyshev)$	118.5414	4.831989	118.0102	112.3726	124.0659
	$DE_F(Circle)$	115.0351	8.039375	114.6847	107.6219	121.2744
	$DE_F(Gauss)$	112.1651	5.525413	112.3409	105.9624	118.8493
	$DE_F(Iterative)$	115.2094	5.374815	115.2919	108.3094	122.7327
	$DE_F(Logistic)$	108.8095	3.24709	109.7967	100.0013	114.4839
	$DE_F(Piecewise)$	114.7536	2.287724	114.9417	110.2476	118.5782
	$DE_F(Sine)$	109.0559	2.945643	109.0521	100	114.1225
	$DE_F(Singer)$	117.9729	4.404494	117.8223	113.3765	124.3794
	$DE_F(Sinusoidal)$	123.4633	4.238903	123.7106	116.5413	130.5993
	$DE_F(Tent)$	113.9102	4.275015	114.6787	107.2742	119.7168
f_{25}	DE	184.7919	27.47824	197.4722	126.9887	201.3282
	$DE_F(Chebyshev)$	175.6954	32.42282	185.6581	129.723	201.2701
	$DE_F(Circle)$	168.1402	23.49974	163.8936	133.4841	201.2932
	$DE_F(Gauss)$	162.8394	57.21216	161.5184	114.6108	201.3759
	$DE_F(Iterative)$	176.6314	25.57945	178.7745	123.5378	201.2977
	$DE_F(Logistic)$	173.8812	37.99965	175.9734	130.774	201.3248
	$DE_F(Piecewise)$	177.6412	40.95452	201.1293	119.3975	201.4974
	$DE_F(Sine)$	176.1857	32.09769	184.7239	124.2041	201.2932
	$DE_F(Singer)$	174.5448	21.69264	177.5088	136.3235	201.2936
	$DE_F(Sinusoidal)$	181.2202	29.35824	186.7922	145.3914	201.438
	$DE_F(Tent)$	176.3676	31.08516	175.6816	146.7405	201.2932
f_{26}	DE	100.1761	0.058563	100.1797	100.1228	100.2423
	$DE_F(Chebyshev)$	100.1918	0.068496	100.1915	100.1135	100.2581
	$DE_F(Circle)$	100.162	0.055355	100.1636	100.1011	100.2246
	$DE_F(Gauss)$	100.1275	0.053694	100.1271	100.066	100.1732
	$DE_F(Iterative)$	100.1724	0.048259	100.1764	100.0709	100.2225
	$DE_F(Logistic)$	100.1621	0.058398	100.1586	100.0897	100.2331
	$DE_F(Piecewise)$	100.1496	0.041843	100.1553	100.0719	100.2079
	$DE_F(Sine)$	100.1559	0.037977	100.1562	100.0794	100.2265
	$DE_F(Singer)$	100.195	0.094053	100.1848	100.1227	100.3069
	$DE_F(Sinusoidal)$	100.2209	0.056394	100.223	100.1516	100.2688
	$DE_F(Tent)$	100.1721	0.058498	100.1686	100.0797	100.2231
f_{27}	DE	195.6459	173.9974	300	2.049225	300.0002
	$DE_F(Chebyshev)$	220.7112	153.5786	300.0005	2.008326	300.3879
	$DE_F(Circle)$	171.4323	195.4923	300	2.042603	303.5865
	$DE_F(Gauss)$	242.2341	148.0023	309.9358	3.248356	332.2091
	$DE_F(Iterative)$	270.2543	94.06537	300	2.33452	300.0103
	$DE_F(Logistic)$	211.151	162.2264	300	2.052592	300.0519
	$DE_F(Piecewise)$	164.4572	231.7244	151.3162	1.84109	400.1357
	$DE_F(Sine)$	194.3451	177.0697	300	1.598971	331.7902
	$DE_F(Singer)$	181.2887	188.018	300.0006	1.804202	305.7786
	$DE_F(Sinusoidal)$	192.8625	238.3312	300.0538	2.084892	320.9548
	$DE_F(Tent)$	220.8006	254.9275	300	2.29312	300.0163



f_{28}	DE	380.0988	19.29837	378.3272	368.8503	471.1331
	$DE_F(\text{Chebysev})$	376.1619	7.290333	378.3272	368.8503	394.0607
	$DE_F(\text{Circle})$	374.5013	8.656806	378.3272	356.8269	380.6274
	$DE_F(\text{Gauss})$	379.4924	8.047879	380.6292	368.9451	404.2403
	$DE_F(\text{Iterative})$	377.1038	5.310784	378.3272	369.3714	380.6276
	$DE_F(\text{Logistic})$	377.1937	9.490901	378.3272	368.8503	390.5749
	$DE_F(\text{Piecewise})$	378.9671	21.92099	378.3272	368.8503	480.8413
	$DE_F(\text{Sine})$	375.5149	6.978469	378.3272	369.3714	380.6274
	$DE_F(\text{Singer})$	377.3299	5.823208	378.3272	368.8503	386.4572
	$DE_F(\text{Sinusoidal})$	377.5826	6.866726	378.3272	369.3714	391.4426
	$DE_F(\text{Tent})$	376.0584	8.167467	378.3272	369.3714	383.9272
f_{29}	DE	311.6286	90.92595	290.7002	234.8726	732.8981
	$DE_F(\text{Chebysev})$	320.1379	77.47198	311.3116	239.9908	426.9912
	$DE_F(\text{Circle})$	314.5155	62.59205	297.5518	251.8356	495.7328
	$DE_F(\text{Gauss})$	493.1463	210.4691	433.6536	284.7243	1346.104
	$DE_F(\text{Iterative})$	316.9578	68.1642	292.4682	242.5224	474.0179
	$DE_F(\text{Logistic})$	341.6384	94.87277	317.9627	248.0133	506.8876
	$DE_F(\text{Piecewise})$	226.3331	20.31006	222.7162	221.5491	332.5332
	$DE_F(\text{Sine})$	328.9427	112.509	281.1259	243.8883	547.293
	$DE_F(\text{Singer})$	325.4346	127.4979	312.9483	250.8799	538.6703
	$DE_F(\text{Sinusoidal})$	367.3944	72.81976	364.5427	253.8974	516.4472
	$DE_F(\text{Tent})$	314.1973	94.52496	295.8132	244.854	583.0799
f_{30}	DE	487.8832	15.75776	487.5098	461.0816	519.9891
	$DE_F(\text{Chebysev})$	483.7287	19.44652	484.2377	459.6285	509.0934
	$DE_F(\text{Circle})$	479.6765	16.16115	476.7917	459.6285	522.0571
	$DE_F(\text{Gauss})$	492.3594	26.16115	491.0113	459.6285	559.5258
	$DE_F(\text{Iterative})$	485.5924	24.02136	486.6163	459.6285	507.283
	$DE_F(\text{Logistic})$	478.2618	26.5221	479.1805	459.6285	551.7125
	$DE_F(\text{Piecewise})$	465.3035	8.624553	462.802	462.307	499.4102
	$DE_F(\text{Sine})$	479.3167	25.73273	481.2552	459.6285	500.9609
	$DE_F(\text{Singer})$	487.5583	15.41595	488.6496	459.6285	516.0548
	$DE_F(\text{Sinusoidal})$	492.1979	20.38843	491.3219	459.6311	527.2939
	$DE_F(\text{Tent})$	485.9396	21.7532	485.8538	459.6285	554.1988

5.2 Maps applied to the scaling factor F

For the unimodal problems, the piecewise map gives by far the best overall result in Table 1: on $f1$ its mean error is $1.49E - 06$ against 5937.349 for DE , whereas the other maps are close to or worse than DE (the singer and sinusoidal maps are clearly worse). On $f2$ and $f3$ all maps except Gauss reach the $1E - 08$ accuracy level, and the Gauss map fails. For the simple multimodal problems, the sine, Gauss and logistic maps perform better than the others; for example, on $f9$ the mean errors of the logistic, sine and Gauss maps are 2.63, 2.33 and 4.50 respectively against 12.82 for DE , and on $f11$ the Gauss map reduces the mean error from 648.78 to 103.62. For the hybrid functions the piecewise and sine maps are good (for example $f17$: 9.91 and 16.51 against 58.35 for DE), while for $f22$ no map improves on DE . The composition functions follow the hybrid functions: the piecewise map is best on $f27$, $f29$ and $f30$, and the logistic and sine maps are best on $f24$.

5.3 Maps applied to the crossover probability CR

For CR (Table 2), the iterative, logistic, sine, singer and sinusoidal maps all reach the $1E-08$ accuracy level on the unimodal function f_1 (the sinusoidal map is therefore among the best, together with these maps), whereas the Gauss map is worse than DE . As for F on the multimodal problems, the logistic, sine and Gauss maps work well for CR (for example f_9 : 6.86, 6.66 and 5.08 against 12.82 for DE). For the hybrid functions the piecewise and iterative maps perform well, and for the composition functions the sine and Gauss maps do.

Table 2: DE results with chaotic crossover probability $CR(DE_{CR})$ on f_1 – f_{30} (MeanE, SD, MD, Best, Worst)

Test problem	Algorithm	MeanE	SD	MD	Best	Worst
f_1	DE	5937.349	2772.828	5408.219	1707.588	12533.67
	$DE_{CR}(Chebysev)$	1.24E-08	1.72E-08	9.4E-09	7.42E-09	1.04E-07
	$DE_{CR}(Circle)$	0.000565	0.000779	0.000284	3.87E-05	0.003116
	$DE_{CR}(Gauss)$	10855.75	4515.068	10445.21	3958.039	19631.25
	$DE_{CR}(Iterative)$	8.02E-09	1.97E-09	8.17E-09	5.47E-09	9.93E-09
	$DE_{CR}(Logistic)$	8.32E-09	1.17E-09	8.25E-09	5.96E-09	9.95E-09
	$DE_{CR}(Piecewise)$	1.57E-07	1.5E-07	1.4E-07	1.48E-08	4.18E-07
	$DE_{CR}(Sine)$	8.43E-09	2.08E-09	9.1E-09	4.77E-09	9.93E-09
	$DE_{CR}(Singer)$	8.39E-09	1.25E-09	8.67E-09	4.92E-09	9.99E-09
	$DE_{CR}(Sinusoidal)$	8.15E-09	2.84E-09	8.54E-09	5.04E-09	9.95E-09
	$DE_{CR}(Tent)$	7.4E-08	1.16E-07	2.34E-08	6.04E-09	4.77E-07
f_2	DE	8.12E-09	2.42E-09	8.41E-09	4.24E-09	9.98E-09
	$DE_{CR}(Chebysev)$	8.43E-09	5.4E-09	8.93E-09	3.13E-09	1E-08
	$DE_{CR}(Circle)$	8.1E-09	2.24E-09	8.83E-09	4.81E-09	9.95E-09
	$DE_{CR}(Gauss)$	3.18E-05	7.7E-05	1.76E-05	2.45E-06	0.000123
	$DE_{CR}(Iterative)$	8.51E-09	1.95E-09	8.85E-09	2.75E-09	9.92E-09
	$DE_{CR}(Logistic)$	8.25E-09	1.43E-09	8.56E-09	4.61E-09	9.97E-09
	$DE_{CR}(Piecewise)$	8.42E-09	1.26E-09	8.57E-09	4.72E-09	9.89E-09
	$DE_{CR}(Sine)$	8.51E-09	1.44E-09	8.62E-09	5.4E-09	9.9E-09
	$DE_{CR}(Singer)$	8.27E-09	2.05E-09	8.78E-09	4.74E-09	9.91E-09
	$DE_{CR}(Sinusoidal)$	7.72E-09	2.27E-09	8.22E-09	3.43E-09	9.66E-09
	$DE_{CR}(Tent)$	8.14E-09	1.25E-09	8.31E-09	3.78E-09	9.69E-09
f_3	DE	7.85E-09	1.66E-09	62506.67	2.53E-09	9.79E-09
	$DE_{CR}(Chebysev)$	8.42E-09	2.11E-09	29166.67	3.77E-09	9.98E-09
	$DE_{CR}(Circle)$	8.24E-09	1.7E-09	42231.67	4.79E-09	9.84E-09
	$DE_{CR}(Gauss)$	5.89E-05	0.000103	100000	3.27E-06	0.000547
	$DE_{CR}(Iterative)$	7.73E-09	1.94E-09	35181.67	2.9E-09	1E-08
	$DE_{CR}(Logistic)$	7.79E-09	2.7E-09	32953.33	3.17E-09	9.91E-09
	$DE_{CR}(Piecewise)$	7.96E-09	1.91E-09	38010	4.23E-09	9.99E-09
	$DE_{CR}(Sine)$	8.04E-09	1.35E-09	33978.33	5.07E-09	9.98E-09
	$DE_{CR}(Singer)$	8.04E-09	1.79E-09	31018.33	3.59E-09	9.96E-09
	$DE_{CR}(Sinusoidal)$	7.55E-09	2.01E-09	32571.67	5.05E-09	9.86E-09



f_4	$DE_{Cr}(Tent)$	8.46E-09	2.08E-09	37986.67	5.05E-09	9.96E-09
	DE	29.2802	62506.67	34.78027	0.437528	34.78027
	$DE_{Cr}(Chebysev)$	25.95661	29166.67	34.78027	0.000163	34.78027
	$DE_{Cr}(Circle)$	20.47758	42231.67	34.78027	9.44E-06	34.78027
	$DE_{Cr}(Gauss)$	10.43996	100000	3.801924	0.632484	34.78027
	$DE_{Cr}(Iterative)$	23.80469	35181.67	34.78027	0.017927	34.78027
	$DE_{Cr}(Logistic)$	28.14461	32953.33	34.78027	0.20077	34.78027
	$DE_{Cr}(Piecewise)$	25.83893	38010	34.78027	0.161455	34.78027
	$DE_{Cr}(Sine)$	23.6799	33978.33	34.78027	0.168225	34.78027
	$DE_{Cr}(Singer)$	23.93449	31018.33	34.78027	0.051403	34.78027
	$DE_{Cr}(Sinusoidal)$	23.65647	32571.67	34.78027	0.094109	34.78027
	$DE_{Cr}(Tent)$	26.25762	37986.67	34.78027	0.278392	34.78027
f_5	DE	20.08192	1.447293	20.14811	18.6632	20.23561
	$DE_{Cr}(Chebysev)$	20.15015	0.062862	20.15793	19.9492	20.2279
	$DE_{Cr}(Circle)$	20.04824	0.587453	20.15891	16.90526	20.24185
	$DE_{Cr}(Gauss)$	20.14338	0.051436	20.14503	20.03817	20.22037
	$DE_{Cr}(Iterative)$	20.11737	0.247275	20.16004	18.84978	20.24319
	$DE_{Cr}(Logistic)$	20.0974	0.288863	20.14668	18.69719	20.21304
	$DE_{Cr}(Piecewise)$	20.15304	0.060218	20.15745	20.09615	20.23111
	$DE_{Cr}(Sine)$	20.14758	0.050582	20.14017	20.07072	20.23842
	$DE_{Cr}(Singer)$	20.11952	0.107398	20.1443	19.85373	20.24064
	$DE_{Cr}(Sinusoidal)$	20.15821	0.05337	20.15344	20.01212	20.22869
	$DE_{Cr}(Tent)$	20.11013	0.280753	20.16301	18.67965	20.2248
f_6	DE	0.20218	0.437053	9.34E-09	4.35E-09	4.35E-09
	$DE_{Cr}(Chebysev)$	0.239436	0.795992	9.05E-09	3.65E-09	3.65E-09
	$DE_{Cr}(Circle)$	0.106124	0.29781	8.61E-09	4.08E-09	4.08E-09
	$DE_{Cr}(Gauss)$	0.03552	0.065424	0.007028	1.29E-05	1.29E-05
	$DE_{Cr}(Iterative)$	0.202143	0.783573	8.91E-09	2.66E-09	2.66E-09
	$DE_{Cr}(Logistic)$	0.077861	0.241515	8.53E-09	4.92E-09	4.92E-09
	$DE_{Cr}(Piecewise)$	0.154572	0.379573	8.84E-09	5.67E-09	5.67E-09
	$DE_{Cr}(Sine)$	0.083465	0.27105	8.72E-09	5.47E-09	5.47E-09
	$DE_{Cr}(Singer)$	0.246805	0.623696	9.16E-09	4.92E-09	4.92E-09
	$DE_{Cr}(Sinusoidal)$	0.212522	0.448185	8.59E-09	4.96E-09	4.96E-09
	$DE_{Cr}(Tent)$	0.08243	0.261728	8.57E-09	6.21E-09	6.21E-09
f_7	DE	0.115159	0.182059	0.104702	0.025318	0.284757
	$DE_{Cr}(Chebysev)$	0.019656	0.029915	0.012319	6.24E-09	0.103552
	$DE_{Cr}(Circle)$	0.052989	0.027679	0.056569	6.29E-09	0.100766
	$DE_{Cr}(Gauss)$	0.018043	0.009928	0.017295	0.002443	0.03387
	$DE_{Cr}(Iterative)$	0.031417	0.035038	0.017242	5.82E-09	0.090909
	$DE_{Cr}(Logistic)$	0.015931	0.021356	0.009857	5.71E-09	0.066428
	$DE_{Cr}(Piecewise)$	0.036056	0.055261	0.03578	6.36E-09	0.095677
	$DE_{Cr}(Sine)$	0.024946	0.027501	0.023069	4.87E-09	0.08075
	$DE_{Cr}(Singer)$	0.019855	0.042423	0.009857	5.56E-09	0.183313
	$DE_{Cr}(Sinusoidal)$	0.060762	0.198599	0.023411	7.43E-09	0.275404



f_8	$DE_{Cr}(Tent)$	0.03312	0.045356	0.021871	4.05E-09	0.116876
	DE	7.83E-09	1.84E-09	8.2E-09	4.44E-09	9.95E-09
	$DE_{Cr}(Chebysev)$	8.37E-09	1.87E-09	8.93E-09	3.89E-09	9.66E-09
	$DE_{Cr}(Circle)$	8.19E-09	2.22E-09	8.88E-09	4.55E-09	9.87E-09
	$DE_{Cr}(Gauss)$	8.51E-09	1.4E-09	8.97E-09	6.08E-09	9.91E-09
	$DE_{Cr}(Iterative)$	7.81E-09	2.57E-09	7.78E-09	5.28E-09	9.8E-09
	$DE_{Cr}(Logistic)$	7.82E-09	1.73E-09	8.15E-09	3.32E-09	9.92E-09
	$DE_{Cr}(Piecewise)$	8.19E-09	3.73E-09	8.65E-09	4.68E-09	9.85E-09
	$DE_{Cr}(Sine)$	7.83E-09	2.28E-09	8.27E-09	4.27E-09	9.97E-09
	$DE_{Cr}(Singer)$	7.96E-09	2.24E-09	8.54E-09	4.15E-09	9.97E-09
	$DE_{Cr}(Sinusoidal)$	0.18747	0.609725	8.9E-09	4.05E-09	2.313623
	$DE_{Cr}(Tent)$	7.91E-09	2.49E-09	8.94E-09	3.07E-09	9.84E-09
f_9	DE	12.81808	2.240374	13.01878	8.56814	16.87345
	$DE_{Cr}(Chebysev)$	10.21022	5.662846	10.26177	5.651618	16.15146
	$DE_{Cr}(Circle)$	8.716038	1.764889	8.542233	5.118921	11.60421
	$DE_{Cr}(Gauss)$	5.077787	2.474658	5.115646	2.376882	6.836083
	$DE_{Cr}(Iterative)$	9.111147	2.101947	9.41388	5.88629	12.21107
	$DE_{Cr}(Logistic)$	6.858311	1.451482	6.751141	3.752024	9.433408
	$DE_{Cr}(Piecewise)$	9.031962	1.566166	9.067705	5.757621	12.34023
	$DE_{Cr}(Sine)$	6.660717	2.737121	6.627378	4.3556	9.150844
	$DE_{Cr}(Singer)$	11.53589	2.831765	11.62497	7.577944	16.78214
	$DE_{Cr}(Sinusoidal)$	16.2302	7.04829	17.44975	2.984883	23.89899
	$DE_{Cr}(Tent)$	8.460709	2.224969	8.706384	4.366231	11.44041
f_{10}	DE	14.97587	14.3969	15.39293	0.249818	33.84444
	$DE_{Cr}(Chebysev)$	13.26074	19.99431	15.22923	0.124909	31.43189
	$DE_{Cr}(Circle)$	11.12642	9.379129	8.707375	0.062454	30.2725
	$DE_{Cr}(Gauss)$	6.663532	10.22944	3.508642	0.124909	35.7331
	$DE_{Cr}(Iterative)$	11.46143	7.437323	11.04897	0.187363	29.38535
	$DE_{Cr}(Logistic)$	8.432725	8.53935	6.988565	0.062454	27.46021
	$DE_{Cr}(Piecewise)$	13.69348	9.57576	14.08285	0.068997	38.9607
	$DE_{Cr}(Sine)$	10.09215	12.17784	11.85771	0.113274	26.97263
	$DE_{Cr}(Singer)$	11.55159	10.74015	11.49887	0.249818	26.38668
	$DE_{Cr}(Sinusoidal)$	23.8524	9.544413	27.13382	6.829921	42.20335
	$DE_{Cr}(Tent)$	12.5166	13.98383	12.28063	0.124909	24.02848
f_{11}	DE	648.7775	163.5534	660.8578	368.1607	852.3998
	$DE_{Cr}(Chebysev)$	631.7146	181.616	631.2709	265.6037	901.7584
	$DE_{Cr}(Circle)$	616.0988	162.0719	614.6866	377.1859	822.659
	$DE_{Cr}(Gauss)$	399.1587	178.9582	417.6874	162.5179	660.3324
	$DE_{Cr}(Iterative)$	634.4109	145.6216	653.7877	376.243	917.052
	$DE_{Cr}(Logistic)$	557.7288	115.97	563.2151	248.3121	772.104
	$DE_{Cr}(Piecewise)$	587.5368	140.0277	611.1196	211.5516	872.201
	$DE_{Cr}(Sine)$	501.7266	158.3129	505.8288	178.1009	755.679
	$DE_{Cr}(Singer)$	596.1558	192.765	600.6928	245.421	825.1737
	$DE_{Cr}(Sinusoidal)$	691.8211	186.933	719.4443	117.0463	1006.321



f_{12}	$DE_{Cr}(Tent)$	578.3199	196.1289	568.6929	351.9857	882.4059
	DE	0.49362	0.089748	0.488117	0.262867	0.656945
	$DE_{Cr}(Chebysev)$	0.500518	0.242151	0.502015	0.303134	0.719623
	$DE_{Cr}(Circle)$	0.491882	0.109019	0.513299	0.26378	0.673468
	$DE_{Cr}(Gauss)$	0.46086	0.105073	0.47435	0.285838	0.722127
	$DE_{Cr}(Iterative)$	0.502634	0.110983	0.502833	0.324966	0.670292
	$DE_{Cr}(Logistic)$	0.498615	0.078099	0.501995	0.290992	0.627003
	$DE_{Cr}(Piecewise)$	0.523865	0.227437	0.530789	0.316952	0.680834
	$DE_{Cr}(Sine)$	0.478651	0.121816	0.481147	0.2848	0.693054
	$DE_{Cr}(Singer)$	0.492343	0.096915	0.501859	0.297169	0.695286
	$DE_{Cr}(Sinusoidal)$	0.469358	0.151776	0.476128	0.2398	0.674863
f_{13}	$DE_{Cr}(Tent)$	0.466468	0.093443	0.456156	0.315845	0.657487
	DE	0.16784	0.033808	0.172403	0.090462	0.218289
	$DE_{Cr}(Chebysev)$	0.124431	0.077796	0.122632	0.04866	0.196753
	$DE_{Cr}(Circle)$	0.142985	0.043948	0.144748	0.085272	0.193259
	$DE_{Cr}(Gauss)$	0.178752	0.040628	0.176604	0.107823	0.261969
	$DE_{Cr}(Iterative)$	0.127507	0.031509	0.131607	0.07921	0.173643
	$DE_{Cr}(Logistic)$	0.132586	0.069319	0.130105	0.050051	0.195893
	$DE_{Cr}(Piecewise)$	0.146721	0.033583	0.146821	0.078725	0.208841
	$DE_{Cr}(Sine)$	0.127659	0.028764	0.129062	0.059808	0.179033
	$DE_{Cr}(Singer)$	0.126555	0.03454	0.125395	0.034219	0.188881
	$DE_{Cr}(Sinusoidal)$	0.147944	0.037758	0.144181	0.064512	0.231789
f_{14}	$DE_{Cr}(Tent)$	0.138455	0.066973	0.144763	0.075469	0.175741
	DE	0.159188	0.04596	0.162751	0.053444	0.234795
	$DE_{Cr}(Chebysev)$	0.139082	0.074312	0.134746	0.071811	0.226915
	$DE_{Cr}(Circle)$	0.163628	0.062663	0.157303	0.066946	0.266815
	$DE_{Cr}(Gauss)$	0.164974	0.044548	0.167496	0.100173	0.22929
	$DE_{Cr}(Iterative)$	0.139299	0.037616	0.143695	0.064804	0.216502
	$DE_{Cr}(Logistic)$	0.136557	0.033886	0.138629	0.068601	0.19793
	$DE_{Cr}(Piecewise)$	0.149819	0.055305	0.132558	0.087692	0.323243
	$DE_{Cr}(Sine)$	0.15375	0.045518	0.153894	0.066853	0.239678
	$DE_{Cr}(Singer)$	0.141166	0.083195	0.138592	0.066934	0.217885
	$DE_{Cr}(Sinusoidal)$	0.128611	0.040283	0.124775	0.076686	0.182947
f_{15}	$DE_{Cr}(Tent)$	0.147034	0.052383	0.148939	0.077133	0.21264
	DE	1.456954	0.594387	1.489207	0.680875	2.059501
	$DE_{Cr}(Chebysev)$	1.270126	0.305381	1.261625	0.810062	1.736993
	$DE_{Cr}(Circle)$	1.169205	0.232942	1.119987	0.807122	1.680383
	$DE_{Cr}(Gauss)$	0.85363	0.121528	0.862764	0.646709	1.042148
	$DE_{Cr}(Iterative)$	1.22855	0.25446	1.255726	0.853032	1.772558
	$DE_{Cr}(Logistic)$	0.990013	0.171455	1.010391	0.639759	1.250709
	$DE_{Cr}(Piecewise)$	1.12444	0.431288	1.148463	0.751038	1.585263
	$DE_{Cr}(Sine)$	0.997113	0.152252	1.006569	0.682847	1.394795
	$DE_{Cr}(Singer)$	1.285433	0.261586	1.320984	0.713969	1.568261
	$DE_{Cr}(Sinusoidal)$	1.677684	0.694885	1.699076	0.783901	2.272101



f_{16}	$DE_{Cr}(Tent)$	1.248071	0.215476	1.312476	0.622867	1.522699
	DE	2.564722	0.29272	2.630437	1.987858	2.943722
	$DE_{Cr}(Chebysev)$	2.464458	0.473369	2.518314	1.706347	2.892221
	$DE_{Cr}(Circle)$	2.402416	0.426445	2.428064	1.683238	2.862207
	$DE_{Cr}(Gauss)$	2.220122	0.232968	2.229319	1.669627	2.652059
	$DE_{Cr}(Iterative)$	2.358962	0.362985	2.404815	1.45906	2.871661
	$DE_{Cr}(Logistic)$	2.383383	0.243336	2.38754	1.913248	2.920734
	$DE_{Cr}(Piecewise)$	2.458836	0.218343	2.499444	1.924241	2.875532
	$DE_{Cr}(Sine)$	2.211567	0.283821	2.221256	1.393658	2.787867
	$DE_{Cr}(Singer)$	2.48245	0.307483	2.49276	1.837319	2.868438
	$DE_{Cr}(Sinusoidal)$	2.641848	0.170119	2.668054	2.194084	2.9418
f_{17}	$DE_{Cr}(Tent)$	2.366977	0.376537	2.447053	1.715045	2.767519
	DE	58.34947	29.75083	58.51735	31.36662	102.0071
	$DE_{Cr}(Chebysev)$	14.09827	115.596	0.809694	7.99E-09	166.7941
	$DE_{Cr}(Circle)$	7.361115	9.063796	5.28781	9.07E-05	29.48622
	$DE_{Cr}(Gauss)$	302.4307	162.9277	266.696	55.60171	573.1417
	$DE_{Cr}(Iterative)$	2.486113	10.21044	0.416286	9.43E-09	16.37654
	$DE_{Cr}(Logistic)$	4.515627	10.47872	0.416286	8.9E-09	49.97299
	$DE_{Cr}(Piecewise)$	5.371876	11.65638	1.667965	2.34E-06	52.36971
	$DE_{Cr}(Sine)$	4.676464	11.87651	0.416286	1.75E-08	51.20715
	$DE_{Cr}(Singer)$	3.674927	9.380758	0.416286	4.44E-09	11.58924
	$DE_{Cr}(Sinusoidal)$	10.30258	32.57527	0.416286	9.45E-09	131.474
f_{18}	$DE_{Cr}(Tent)$	2.197968	3.322904	0.624759	0.051655	11.38127
	DE	1.712898	0.607489	1.707726	0.770484	3.006057
	$DE_{Cr}(Chebysev)$	0.494601	0.963018	0.402527	0.00636	2.457754
	$DE_{Cr}(Circle)$	0.497629	0.351849	0.477644	0.017791	1.837625
	$DE_{Cr}(Gauss)$	3.187325	2.4695	3.137304	0.808944	5.421064
	$DE_{Cr}(Iterative)$	0.368327	0.191675	0.388988	0.008907	1.037821
	$DE_{Cr}(Logistic)$	0.51607	0.363198	0.454876	0.024322	1.492317
	$DE_{Cr}(Piecewise)$	0.448791	0.434461	0.341671	0.035381	2.151971
	$DE_{Cr}(Sine)$	0.422191	0.576746	0.344778	0.006899	3.434675
	$DE_{Cr}(Singer)$	0.49436	0.948316	0.396174	0.022688	1.49273
	$DE_{Cr}(Sinusoidal)$	0.311923	0.413869	0.210089	0.000822	1.492057
f_{19}	$DE_{Cr}(Tent)$	0.376874	0.236404	0.385824	0.010526	1.265962
	DE	0.270101	0.20881	0.247478	0.072506	0.584808
	$DE_{Cr}(Chebysev)$	0.223305	0.222846	0.143326	0.02685	0.819253
	$DE_{Cr}(Circle)$	0.294791	0.30764	0.201277	0.046802	0.972684
	$DE_{Cr}(Gauss)$	0.279536	0.139305	0.277248	0.127415	0.567914
	$DE_{Cr}(Iterative)$	0.204599	0.170247	0.144554	0.057915	0.757379
	$DE_{Cr}(Logistic)$	0.149237	0.158101	0.113331	0.026828	0.775965
	$DE_{Cr}(Piecewise)$	0.178925	0.141968	0.1494	0.011511	0.557139
	$DE_{Cr}(Sine)$	0.279101	0.266862	0.159762	0.058671	0.998345
	$DE_{Cr}(Singer)$	0.321853	0.59137	0.21848	0.00241	1.246523
	$DE_{Cr}(Sinusoidal)$	0.485887	0.346625	0.319044	0.08559	1.29779



f_{20}	$DE_{Cr}(Tent)$	0.218017	0.194691	0.143829	0.079524	0.941407
	DE	0.117829	0.142417	0.092674	0.001166	0.476347
	$DE_{Cr}(Chebysev)$	0.057086	0.130363	0.003377	7.15E-06	0.480938
	$DE_{Cr}(Circle)$	0.070707	0.141501	0.019739	2.09E-05	0.487137
	$DE_{Cr}(Gauss)$	0.142798	0.125662	0.128283	0.010497	0.443382
	$DE_{Cr}(Iterative)$	0.047706	0.104289	0.005278	4E-05	0.464286
	$DE_{Cr}(Logistic)$	0.145425	0.166639	0.058223	3.41E-05	0.499558
	$DE_{Cr}(Piecewise)$	0.052091	0.102939	0.009992	1.01E-05	0.436998
	$DE_{Cr}(Sine)$	0.081188	0.14392	0.021279	0.000144	0.477369
	$DE_{Cr}(Singer)$	0.060095	0.139001	0.003345	2.04E-07	0.490396
	$DE_{Cr}(Sinusoidal)$	0.086554	0.172687	0.008677	5.91E-06	0.605837
f_{21}	$DE_{Cr}(Tent)$	0.1021	0.183365	0.008193	3.56E-06	0.499178
	DE	0.312439	0.284015	0.32769	0.047756	0.677014
	$DE_{Cr}(Chebysev)$	0.511221	0.644138	0.493976	0.001247	1.122427
	$DE_{Cr}(Circle)$	0.333827	0.67047	0.306898	0.001516	1.087498
	$DE_{Cr}(Gauss)$	1.355317	1.047046	1.013994	0.22621	4.777119
	$DE_{Cr}(Iterative)$	0.893436	3.053757	0.322874	0.000182	16.77404
	$DE_{Cr}(Logistic)$	0.454651	0.283524	0.495667	1.78E-05	0.812169
	$DE_{Cr}(Piecewise)$	0.314859	0.258348	0.31744	0.000662	0.806809
	$DE_{Cr}(Sine)$	0.477121	0.730213	0.499005	0.000397	1.124342
	$DE_{Cr}(Singer)$	0.392115	0.277727	0.462448	0.000144	0.922818
	$DE_{Cr}(Sinusoidal)$	0.497966	0.356351	0.440746	0.001244	1.124333
f_{22}	$DE_{Cr}(Tent)$	0.491371	0.362852	0.497218	0.000812	1.123224
	DE	0.003328	0.00547	0.002121	9.29E-05	0.023571
	$DE_{Cr}(Chebysev)$	0.033877	0.099403	0.001862	7.22E-06	0.319655
	$DE_{Cr}(Circle)$	0.066402	0.160858	0.00199	8.53E-09	0.625124
	$DE_{Cr}(Gauss)$	0.132949	2.431973	0.040207	0.010877	2.523084
	$DE_{Cr}(Iterative)$	0.068452	0.137282	0.004567	1.65E-06	0.319471
	$DE_{Cr}(Logistic)$	0.076665	0.152392	0.002955	6.33E-06	0.320232
	$DE_{Cr}(Piecewise)$	0.0474	0.116905	0.001937	0.000116	0.326681
	$DE_{Cr}(Sine)$	0.118003	0.207695	0.003499	0.000147	0.629267
	$DE_{Cr}(Singer)$	0.127336	0.214564	0.004646	5.13E-05	0.627029
	$DE_{Cr}(Sinusoidal)$	0.117642	0.206909	0.004162	3.54E-05	0.62435
f_{23}	$DE_{Cr}(Tent)$	0.086835	0.179172	0.003668	2.55E-06	0.627965
	DE	329.4575	0	329.4575	329.4575	329.4575
	$DE_{Cr}(Chebysev)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_{Cr}(Circle)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_{Cr}(Gauss)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_{Cr}(Iterative)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_{Cr}(Logistic)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_{Cr}(Piecewise)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_{Cr}(Sine)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_{Cr}(Singer)$	329.4575	0	329.4575	329.4575	329.4575
	$DE_{Cr}(Sinusoidal)$	329.4575	0	329.4575	329.4575	329.4575



f_{24}	$DE_{Cr}(Tent)$	329.4575	0	329.4575	329.4575	329.4575
	DE	119.9061	3.33197	119.7548	115.6218	127.7835
	$DE_{Cr}(Chebysev)$	115.0531	5.320807	115.2965	107.4931	124.5849
	$DE_{Cr}(Circle)$	114.9767	2.285834	114.7896	111.1382	119.3988
	$DE_{Cr}(Gauss)$	113.1971	1.725701	113.4575	109.4004	116.3154
	$DE_{Cr}(Iterative)$	115.8935	3.4776	116.1813	111.4274	121.0343
	$DE_{Cr}(Logistic)$	113.4302	2.228063	113.4184	105.5975	117.8841
	$DE_{Cr}(Piecewise)$	115.4788	1.841046	115.3161	111.5003	120.0492
	$DE_{Cr}(Sine)$	113.3281	5.294601	113.7249	107.8077	117.8296
	$DE_{Cr}(Singer)$	115.9337	11.23297	116.3772	105.4255	123.059
	$DE_{Cr}(Sinusoidal)$	122.9793	9.332095	126.1593	107.3809	131.2485
	$DE_{Cr}(Tent)$	116.0259	2.822438	116.0561	112.2276	119.467
f_{25}	DE	184.7919	27.47824	197.4722	126.9887	201.3282
	$DE_{Cr}(Chebysev)$	176.7587	44.96057	201.1293	100	201.3603
	$DE_{Cr}(Circle)$	165.8663	49.53619	168.3556	114.8003	201.2932
	$DE_{Cr}(Gauss)$	156.1998	22.41509	150.0228	131.2076	201.3705
	$DE_{Cr}(Iterative)$	171.2643	48.03756	201.1293	100.6235	201.3195
	$DE_{Cr}(Logistic)$	184.447	36.7263	201.1293	104.1175	201.3355
	$DE_{Cr}(Piecewise)$	161.0028	51.7721	155.1677	120.5062	201.2932
	$DE_{Cr}(Sine)$	154.9173	60.80934	138.5506	100	201.3828
	$DE_{Cr}(Singer)$	161.2815	58.02235	161.4441	109.1962	201.2969
	$DE_{Cr}(Sinusoidal)$	167.7203	54.00072	195.1157	108.7147	201.3195
	$DE_{Cr}(Tent)$	162.0487	54.85789	155.8792	112.2964	201.3197
f_{26}	DE	100.1761	0.058563	100.1797	100.1228	100.2423
	$DE_{Cr}(Chebysev)$	100.1207	0.044476	100.1293	100.0356	100.1742
	$DE_{Cr}(Circle)$	100.1546	0.046048	100.1499	100.0986	100.2148
	$DE_{Cr}(Gauss)$	100.185	0.042225	100.1906	100.1191	100.2608
	$DE_{Cr}(Iterative)$	100.1496	0.041867	100.1516	100.0879	100.2135
	$DE_{Cr}(Logistic)$	100.1384	0.028592	100.1398	100.0825	100.2037
	$DE_{Cr}(Piecewise)$	100.1452	0.028273	100.1464	100.0971	100.1974
	$DE_{Cr}(Sine)$	100.1419	0.032745	100.1464	100.0951	100.1934
	$DE_{Cr}(Singer)$	100.1392	0.064072	100.1411	100.0795	100.1811
	$DE_{Cr}(Sinusoidal)$	100.1424	0.047272	100.1477	100.051	100.2231
	$DE_{Cr}(Tent)$	100.1481	0.03223	100.1477	100.0804	100.2151
f_{27}	DE	195.6459	173.9974	300	2.049225	300.0002
	$DE_{Cr}(Chebysev)$	177.5477	242.9549	300	1.280983	400.1712
	$DE_{Cr}(Circle)$	157.7707	221.2192	151.3303	1.738216	400.1326
	$DE_{Cr}(Gauss)$	169.9008	225.4752	303.2931	3.207618	345.7377
	$DE_{Cr}(Iterative)$	190.9149	268.9738	300	1.506582	400.1514
	$DE_{Cr}(Logistic)$	204.2836	204.4477	300	1.297565	400.1675
	$DE_{Cr}(Piecewise)$	157.712	221.5847	151.344	1.690679	400.1317
	$DE_{Cr}(Sine)$	144.5168	218.469	2.331657	1.293371	400.1357
	$DE_{Cr}(Singer)$	221.5212	166.2766	300	1.474236	400.1514
	$DE_{Cr}(Sinusoidal)$	247.1851	162.4343	300	1.203083	400.136

f_{28}	$DE_{Cr}(Tent)$	177.6397	207.6113	300	1.511123	400.1332
	DE	380.0988	19.29837	378.3272	368.8503	471.1331
	$DE_{Cr}(Chebysev)$	375.1634	9.809957	375.2205	356.8269	404.1861
	$DE_{Cr}(Circle)$	376.9841	6.303824	378.3272	368.8503	386.0109
	$DE_{Cr}(Gauss)$	373.3767	8.933303	372.537	357.9355	380.6279
	$DE_{Cr}(Iterative)$	377.4601	98.16454	375.2205	368.8503	473.8658
	$DE_{Cr}(Logistic)$	378.7924	13.93515	378.3272	368.8503	418.119
	$DE_{Cr}(Piecewise)$	376.4239	8.245306	378.3272	368.8503	394.1193
	$DE_{Cr}(Sine)$	374.7106	5.532074	376.4563	368.8503	386.4572
	$DE_{Cr}(Singer)$	376.0505	5.290347	378.3272	368.8503	380.8277
	$DE_{Cr}(Sinusoidal)$	377.5422	10.63892	378.4987	368.8503	392.0509
	$DE_{Cr}(Tent)$	380.409	20.3104	378.3272	369.3714	485.9958
f_{29}	DE	311.6286	90.92595	290.7002	234.8726	732.8981
	$DE_{Cr}(Chebysev)$	221.8895	4.007212	223.1281	206.3887	224.8238
	$DE_{Cr}(Circle)$	222.8673	1.447237	223.2565	221.5554	224.8884
	$DE_{Cr}(Gauss)$	446.0317	231.5996	414.6457	266.89	1570.295
	$DE_{Cr}(Iterative)$	273.43	278.5262	222.4291	221.5407	1747.294
	$DE_{Cr}(Logistic)$	222.7226	1.579215	222.5955	221.5407	226.7662
	$DE_{Cr}(Piecewise)$	222.93	1.37233	222.9523	221.5412	225.3369
	$DE_{Cr}(Sine)$	223.1166	1.653523	222.9085	221.5407	230.1481
	$DE_{Cr}(Singer)$	222.6131	1.282274	222.5854	221.5407	225.2944
	$DE_{Cr}(Sinusoidal)$	222.5828	1.516411	222.3732	221.5407	225.2944
	$DE_{Cr}(Tent)$	222.5141	1.146705	222.7176	221.5419	224.5349
f_{30}	DE	487.8832	15.75776	487.5098	461.0816	519.9891
	$DE_{Cr}(Chebysev)$	478.8365	31.99329	462.8746	462.3068	577.6861
	$DE_{Cr}(Circle)$	473.1936	21.02247	463.2275	462.3967	529.9258
	$DE_{Cr}(Gauss)$	493.6386	22.24908	490.9607	467.4602	579.5083
	$DE_{Cr}(Iterative)$	466.1919	10.87074	462.6581	459.6285	499.5182
	$DE_{Cr}(Logistic)$	469.4336	15.61234	462.6458	462.3068	512.9493
	$DE_{Cr}(Piecewise)$	467.9508	13.04817	462.7956	462.3069	500.3926
	$DE_{Cr}(Sine)$	469.025	18.41355	462.7685	462.3068	546.5064
	$DE_{Cr}(Singer)$	505.5802	171.713	463.2982	462.3068	1392.221
	$DE_{Cr}(Sinusoidal)$	467.3914	11.39121	462.7612	462.3068	498.2621
	$DE_{Cr}(Tent)$	467.8634	34.04632	462.7749	462.307	501.6213

5.4 Boxplot analysis

Boxplots of the mean error over all test functions are plotted for F and for CR in figs. 11 and 12, and boxplots of the best values are shown in Figs. 13 and 14. In Fig. 13 (best values, F), the piecewise map attains most of its values in the lower quartile range, i.e. it reaches lower errors than the other maps. The logistic and iterative maps perform comparably but slightly worse than the piecewise map, and the original DE shows a balanced interquartile range. The Gauss map performs worst because of its high upper quartile range. In terms of the interquartile range of the best values, the piecewise map has the widest range, followed by the sine, logistic and Chebyshev maps. Boxplots are descriptive; they are not statistical tests, and a formal test (e.g. the Wilcoxon signed-rank or Friedman test) is recommended to

confirm the differences.

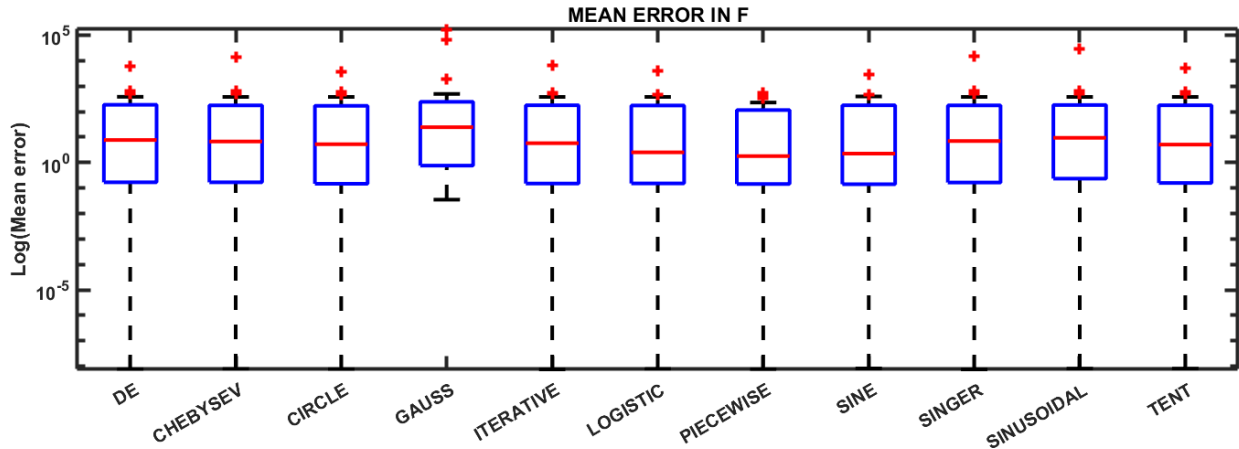


Figure 11

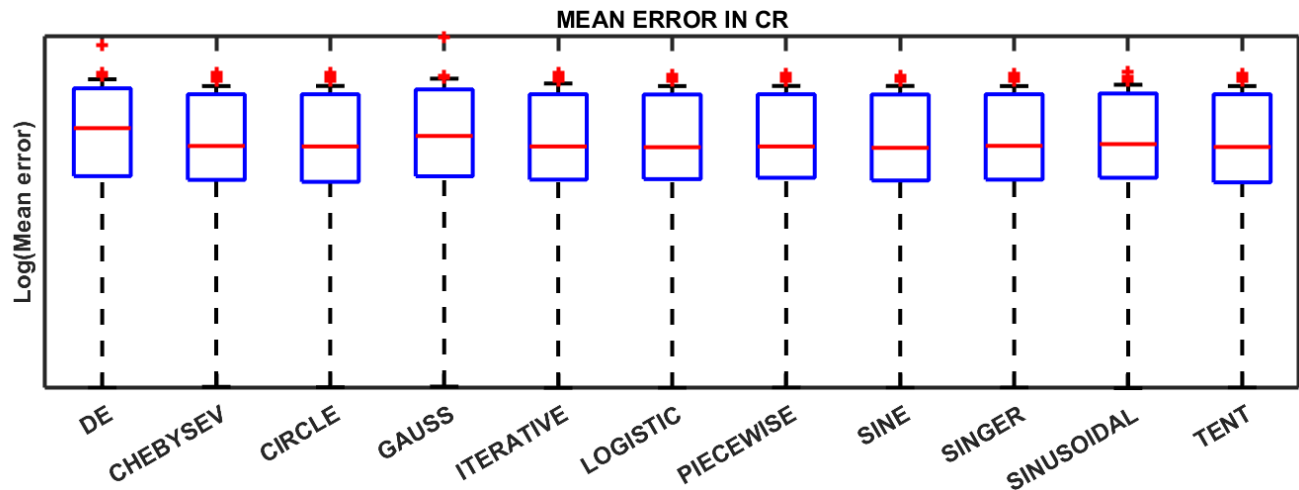


Figure 12

6. Conclusion

In this work we replaced the fixed scaling factor F and crossover probability CR of Differential Evolution with values taken from ten chaotic maps. A diversity check on the population decided when the chaotic values were used. We tested each map on F and on CR separately, using the 30 CEC 2014 functions, and compared every variant with DE run at $F = 0.5$ and $CR = 0.4$. The main thing we learned is that a map's usefulness depends on the kind of problem and on which parameter it controls. When applied to F , the piecewise map gave the biggest gain on the unimodal function $f1$. It also did better than DE on several hybrid and composition functions, for example $f17$, $f18$, $f27$, $f29$ and $f30$. The logistic and sine maps helped most on multimodal functions such as $f9$. The Gauss map gave the lowest errors on $f11$, $f15$ and $f16$, but it failed badly on $f2$, $f3$ and $f18$. The sinusoidal map on F was worse than DE on many functions, and on $f22$ no map on F did better than standard DE .

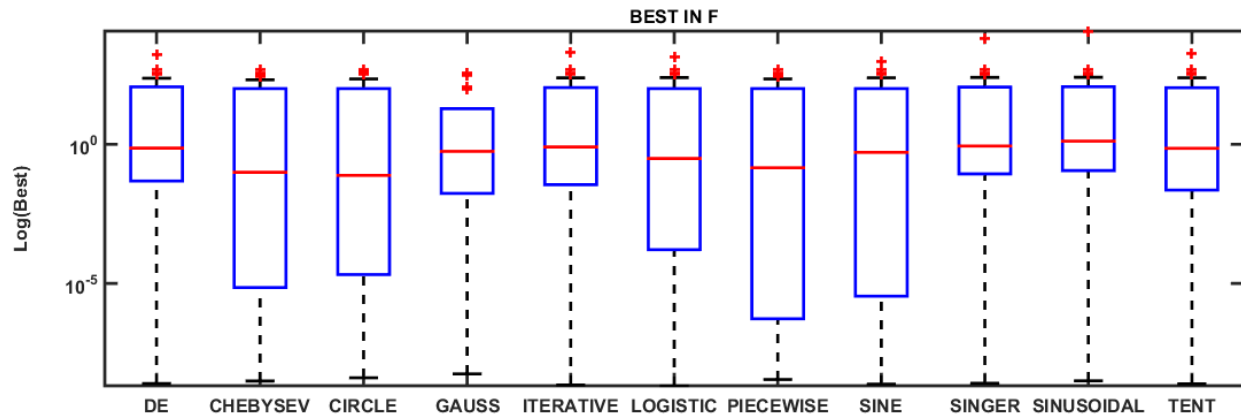


Figure 13

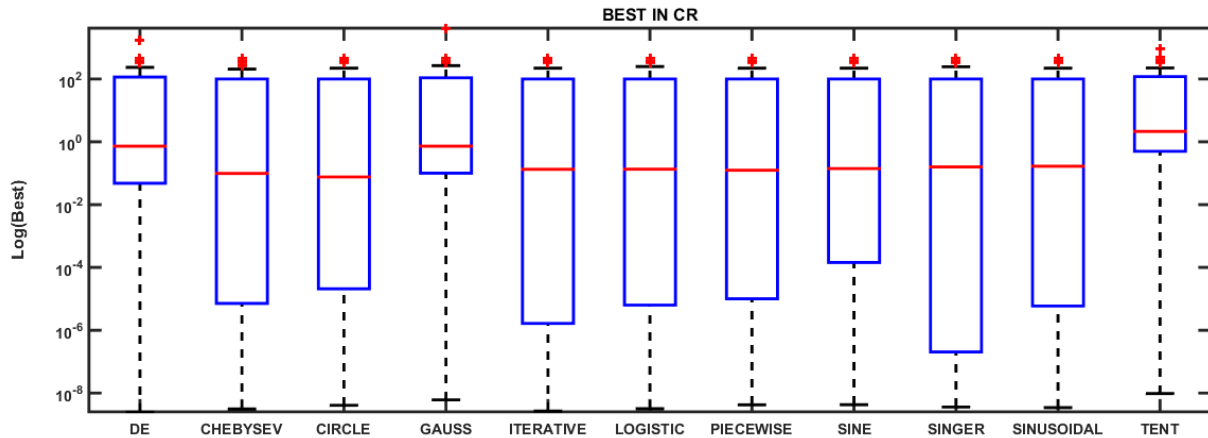


Figure 14

For CR the picture was different. The iterative, logistic, sine, Singer and sinusoidal maps all brought $f1$ down, while the Gauss map did not. The Gauss, sine and logistic maps lowered the error on multimodal problems, and the iterative and tent maps did well on $f17$. On $f29$ most CR maps brought the mean error against for DE . These results suggest that a map worth using on F is not always the right one for CR , and that the choice should follow the problem type. Our study has limits. We compared maps through error tables only and did not run a statistical test, so we cannot say which differences are significant. We also used one fixed setting of F and CR as the baseline, and a better-tuned DE might narrow some of the gaps. In future work we plan to add Wilcoxon or Friedman tests, compare against adaptive DE variants such as JADE and SHADE, try other mutation strategies, test higher dimensions, and let the algorithm choose its chaotic map during the run.

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