

# Analytic Behaviour of a Parallel Redundant Complex System Involving Environmental Failure Under Pre-Emptive-Resume Repair Discipline

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**Dr. Tripti Dixit**

Department of Mathematics

Govt. Post Graduate College Kotdwar Uttarakhand

drtdixit22@gmail.com

## Abstract

In these days reliability requirement is more important. The present paper describes the analytic behaviour of a parallel redundant complex system involving environmental failure under pre-emptive resume repair discipline. In this analysis laplace transform and supplementary variable techniques have been used to find the probabilities of the system being in various states.

## Keywords

Reliability, MTTF, Parallel Redundant, Environmental Failure, Complex System, MTSE, Pre-Emptive, Resume Repair.

## 1. Introduction

In this paper we initiated the study of analytic behaviour of a parallel redundant complex system involving environmental failure under pre-emptive resume repair discipline. This techniques shows a failed component of subsystem  $L_1$  goes to repair immediately on its failure. This means, components of subsystem  $L_1$  pre-emptive repair facilities even when these facilities are engaged in the repair of the failed component of subsystem  $L_2$ . However when the components of subsystem A are taken back in repair facilities, the repair starts from the point where it was left earlier.

## 2. Notations

To formulate the mathematical model, we define the following in additions to the definition of  $P_{0.M}(t)$ ,  $P_{0.M-j}(y, t)$ ,  $P_{F_1.M}(x, t)$ ,  $P_{F_1.M-r}(x, t)$ ,  $P_F(u, t)$ .  $Q_{F_1.M-r}(x, y, t)\Delta$ - The probability that at time 't'. the system is in the failed state due to the failure of the  $i^{\text{th}}$  component of subsystem  $L_1$  and the elapsed repair times lies in the interval  $(x, x + \Delta)$  and at the instant when it pre-empted in the repair facilities, the elapsed time of class  $L_2$  was in the interval  $(y, y + \Delta)$

2.1 Mathematical Model of the Problem

$$[\delta / \delta t + \lambda + v + M\lambda'] P_{0M}(t) = \int_0^\infty P_{0,M-j}(y, t) \phi_j(y) dy + \sum_{i=t}^N \int_0^\infty P_{F_i M}(x, t) \eta_1(x) dx + \int_0^\infty \delta(u) \cdot \bar{P}(u, t) du \dots\dots\dots(1)$$

$$\left[ \frac{\delta}{\delta t} + \lambda + (M - r)\lambda' + v \right] P_{0M-r}(t) = (M - r + 1)\lambda' P_{0,M-r+1}(t) + \sum_{i=1}^N \int_0^\infty P_{F_i M-r}(x, t) \eta_1(x) dx \dots\dots\dots(2)$$

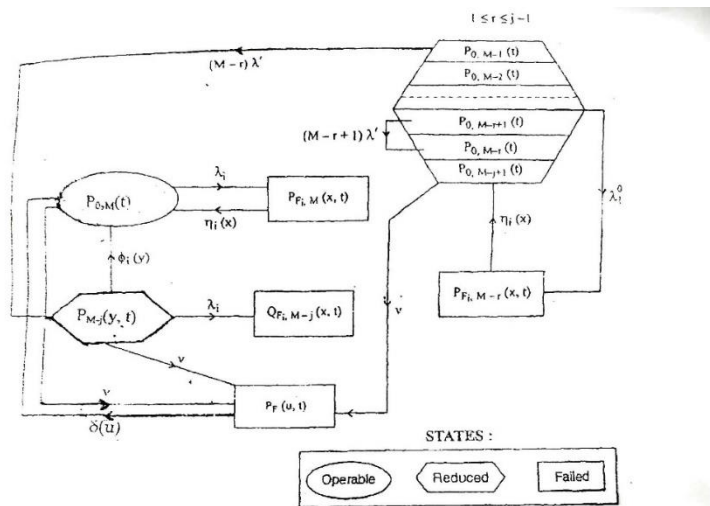
$$\left[ \frac{\delta}{\delta t} + \frac{\delta}{\delta t} + \lambda + v + \phi_j(y) \right] P_{0,M-j}(y, t) = \sum_{i=1}^N \int_0^\infty Q_{F_i M-j}(x, y, t) \eta_1(x) dx \dots\dots\dots(3)$$

$$\left[ \frac{\delta}{\delta t} + \frac{\delta}{\delta t} + \eta_1(x) \right] P_{F_i M}(x, t) = 0 \dots\dots\dots(4)$$

$$\left[ \frac{\delta}{\delta x} + \frac{\delta}{\delta t} + \eta_1(x) \right] P_{F_i M-r}(x, t) = 0 \dots\dots\dots(5)$$

$$\left[ \frac{\delta}{\delta x} + \frac{\delta}{\delta t} + \phi(y) \right] Q_{F_i M-j} \dots\dots\dots(6)$$

$$\left[ \frac{\delta}{\delta u} + \frac{\delta}{\delta t} + \delta(u) \right] P_F(u, t) \dots\dots\dots(7)$$



Initial Conditions

$$P_{0,M}(0) = 1 \dots\dots\dots(8)$$

and other states of various probabilities are zero.

Boundary Conditions

$$P_{F_1 M}(0, t) = \lambda_1 P_{0M}(t) \dots\dots\dots(9)$$

$$P_{F_{M-r}}(0, t) = \lambda P_{0,M-t}(t) \dots\dots\dots(10)$$

$$P_{0M-j}(0, t) = (M - j + 1) \lambda' P_{0,M-j+1}(t) \dots\dots\dots(11)$$

$$Q_{F,M-j}(0, y, t) = \lambda_i \cdot P_{0,M-j}(y, t)$$

.....(12)

$$\bar{P}_f(0, t) = v \left[ \int P_{0M-j}(y, t) dy + P_{0,M-r}(t) + P_{0,M}(t) \right]$$

.....(13)

## 2.2 Solution for the Model

Taking Laplace transform of equation (1) through (13) w.r.t 't' one may obtain,  $(s + \lambda + v + M\lambda')$   $\bar{P}_{0,M}(s) = 1 + \int_0^\infty \bar{P}_{0,M-j}(y, s) \phi_j(y) dy$

$$+ \sum_{i=1}^N \int_0^\infty \bar{P}_{F_i,M}(x, s) \eta_1(x) dx + \int_0^\infty \delta(u) \cdot \bar{P}_f(u, s) \cdot du$$

.....(14)

$$(s + \lambda + \overline{M - r\lambda'} + v) \bar{P}_{0,M-r}(s) + \sum_{i=1}^N \int_0^\infty \bar{P}_{F_i,M}(x, s) \eta_1(x) dx$$

.....(15)

$$\left[ \frac{\partial}{\partial y} + s + \lambda + v + \phi_j(y) \right] \bar{P}_{0,M-r}(y, s) = \sum_{i=1}^N \bar{Q}_{F_i,M-j}(x, y, s) \eta_1(x) dx$$

.....(16)

$$\left[ \frac{\partial}{\partial x} + s + \eta_1 + (x) \right] \bar{P}_{0,M-r}(y, s) = 0$$

.....(17)

$$\left[ \frac{\partial}{\partial x} + s + \eta_1 + (x) \right] \bar{P}_{F M-j}(x, s) = 0$$

.....(18)

$$\left[ \frac{\partial}{\partial x} + s + \eta_1 + (x) \right] \bar{Q}_{F M-j}(x, y, s) = 0$$

.....(19)

$$\left[ \frac{\partial}{\partial x} + s + \delta(u) \right] \bar{P}_f(u, s) = 0$$

.....(20)

$$\bar{P}_{F M-j}(0, s) = \lambda_1 \bar{P}_{0M}(s)$$

.....(21)

$$\bar{P}_{F M-r}(0, s) = \lambda_1 \bar{P}_{0 M-r}(s)$$

.....(22)

$$\bar{P}_{0 M-j}(0, s) = (M - j + 1) \lambda' \bar{P}_{0,M-j+1}(s)$$

.....(23)

$$Q_{F,M-j}(0, y, s) = \lambda_1 \cdot \bar{P}_{0,M-1}(y, s)$$

.....(24)

$$\bar{P}_0(0, s) = v \left[ \int P_{0M-j}(y, s) dy + P_{0M-j}(s) + P_{0M}(s) \right]$$

.....(25)

$$\bar{P}_{F_iM}(x, s) = \lambda_i \bar{P}_{0M}(s) e^{-sx - \int_0^x \eta_1(x) dx}$$

.....(26)

$$\bar{P}_{F_iM}(x, s) = \lambda_i \bar{P}_{0M}(s) e^{-sx - \int_0^x \eta_1(x) dx}$$

.....(27)

$$\bar{Q}_{F_1M-j}(x, y, s) = \lambda' \bar{P}_{0M-j}(y, s) e^{-sx - \int_0^x \eta_1(x) dx}$$

.....(28)

$$\bar{P}_{0M-r}(s) = \frac{(M-r+1)\lambda' \bar{P}_{0M-r+1}(s)}{[s + \lambda + v + (M-r)\lambda' - \sum \lambda_i \bar{S}_i(s)]}$$

.....(29)

$$\bar{P}_{0M-j}(y - s) = (M - r + 1)\lambda' \bar{P}_{0M-j+1}(s) e^{-[s + \lambda + v - \sum \lambda_i \bar{S}_i(s)] y} \times e^{\int_0^x \phi_j(y) dy}$$

.....(30)

$$\bar{P}_F(u, s) = v \left[ \int_0^\infty P_{0M-j}(y, s) dy + \bar{P}_{0M-r}(s) + \bar{P}_{0M}(s) \right] e^{-su - \int_0^u \delta(u) du}$$

.....(31)

$$\bar{P}_{0M}(s) = \frac{1}{L(s)}$$

.....(32)

Where,  $L(s) = s + \lambda + v + M\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s) - (M - j + 1)(\lambda')^j$

$$\begin{aligned} & \times S_j \left\{ s + \lambda + v - \sum_{i=1}^N \lambda_i \bar{S}_i(s) \right\} \times \prod_{r=1}^{j-1} \left( \frac{M - r + 1}{s + \lambda + v + M - r\lambda' - \sum \lambda_i \bar{S}_i(s)} \right) \\ & - v \bar{S}_\delta(s) [(M - j + 1)(\lambda')^j \cdot \bar{S}_j(s + \lambda + v - \sum_{i=1}^N \lambda_i \bar{S}_j(s)) \\ & \quad \times \prod_{r=1}^{j-1} \left( \frac{M - r + 1}{s + \lambda + v + M - r\lambda' - \sum \lambda_i \bar{S}_i(s)} \right) \\ & \quad \times \prod_{P=0}^{j-1} \left( \frac{(\lambda')^t (M - P)}{s + \lambda + v + (M - P - 1)\lambda' - \sum \lambda_i \bar{S}_i(s)} + 1 \right) \end{aligned}$$

$$\bar{P}_{0M-r}(s) = \frac{(\lambda')^r}{L(s)} \cdot \prod_{P=0}^{j-1} \left( \frac{M - P}{s + \lambda + v + M - P - 1\lambda' - \sum \lambda_i \bar{S}_i(s)} \right)$$

.....(33)

$\bar{P}_R(s) = \bar{P}_{0M-j}(s)$ , can be written as

$$\begin{aligned} \bar{P}_R(s) &= \int_0^\infty \bar{P}_{0M-j}(s) dy \\ \bar{P}_R(s) &= \frac{1 - \bar{S}_i\{s + \lambda + v - \sum \lambda_i \bar{S}_i(s)\}}{s + \lambda + v - \sum \lambda_i \bar{S}_i(s)} \cdot \frac{(M - j + 1)(\lambda')^j}{L(s)} \end{aligned}$$

$$\times \prod_{r=1}^{j-1} \left( \frac{(M-r+1)}{s+\lambda+v+M-r\lambda'-\sum \lambda_i \bar{S}_i(s)} \right)$$

.....(34)

$$\bar{P}_F(s) = \frac{\lambda_i}{L(s)} \cdot \frac{1-\bar{S}_i(s)}{s} \tag{35}$$

$$\bar{P}_{F_{iM-r}}(s) = \frac{\lambda_i}{L(s)} \cdot \frac{1-\bar{S}_i(s)}{s} + (\lambda')^r \times \prod_{p=0}^{r-1} \left( \frac{M-p}{s+\lambda+v+M-p-1\lambda'-\sum \lambda_i \bar{S}_i(s)} \right) \tag{36}$$

$$\bar{Q}_{F_{iM-j}}(s) = \frac{\lambda_i}{L(s)} \cdot \frac{1-\bar{S}_i(s)}{s} + \frac{1-\bar{S}_i\{s+\lambda+v-\sum \lambda_i \bar{S}_i(s)\}}{s+\lambda+v-\sum \lambda_i \bar{S}_i(s)} \cdot (M-j+1)(\lambda')^j$$

$$\times \prod_{r=1}^{j-1} \left( \frac{M-r+1}{s+\lambda+v+M-r\lambda'-\sum \lambda_i \bar{S}_i(s)} \right)$$

.....(37)

$$\bar{P}_F(s) = \frac{v\{1-\bar{S}_\delta(s)\}}{L(s) \cdot s} \left[ \frac{1-\bar{S}_i\{s+\lambda+v-\sum \lambda_i \bar{S}_i(s)\}}{s+\lambda+v-\sum \lambda_i \bar{S}_i(s)} (M-j+1)(\lambda')^j \right.$$

$$\times \prod_{r=1}^{j-1} \left( \frac{M-r+1}{s+\lambda+v+M-r\lambda'-\sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right)$$

$$\left. + (\lambda')^r \times \prod_{p=0}^{r-1} \left[ \left( \frac{M-p}{s+\lambda+v+(M-p)\lambda'-\sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right) + 1 \right] \right] \tag{38}$$

### 2.3 Evaluation of Up and Down State Probabilities

$$\bar{P}_{up}(s) = \bar{P}_{0.M}(s) + \sum_{r=1}^{j-1} \bar{P}_{0.M-r}(s) + \bar{P}_r(s)$$

$$= \frac{1}{L(s)} \left[ 1 + \sum_{r=1}^{j-1} \{(\lambda')^r \prod_{r=1}^{r-1} \left( \frac{M-p}{s+\lambda+v+M-p-1\lambda'-\sum \lambda_i \bar{S}_i(s)} \right) \right] p$$

$$+ \frac{1-\bar{S}_i\{s+\lambda+v-\sum \lambda_i \bar{S}_i(s)\}}{s+\lambda+v-\sum \lambda_i \bar{S}_i(s)} \times (M-j+1)(\lambda')^j$$

$$\times \prod_{r=0}^{j-1} \left( \frac{(M-r+1)}{s+\lambda+v+(M-p)\lambda'-\sum_{i=1}^N \lambda_i \bar{S}_i(s)} \right)$$

.....(39)

$$\bar{P}_{down}(s) = \sum_{i=1}^N \left[ \int_0^\infty \bar{P}_{F_{iM}}(x,s) dx + \sum_{r=1}^{j-1} \bar{P}_{F_{iM-r}}(x,s) dx \right.$$

$$\left. + \int_0^\infty Q_{F_{iM-j}}(y,s) dy \right] + \int_0^\infty \bar{P}_F(u,s) du$$

$$= \sum_{i=1}^N \left\{ \frac{\lambda_i}{L(s)} \cdot \frac{1-\bar{S}_i(s)}{s} \right\} \left[ 1 + (\lambda')^r \times \prod_{p=0}^{r-1} \left( \frac{M-p}{s+\lambda+v+M-p-1\lambda'-\sum \lambda_i \bar{S}_i(s)} \right) \right] +$$

$$(M-j+1)(\lambda')^j \cdot \frac{1-\bar{S}_i\{s+\lambda+v-\sum \lambda_i \bar{S}_i(s)\}}{s+\lambda+v-\sum \lambda_i \bar{S}_i(s)}$$

$$\begin{aligned} & \times \prod_{r=1}^{j-1} \left( \frac{M-r+1}{s+\lambda+v+(M-r)\lambda'\bar{S}_1(s)} \right) \\ & + \frac{v.1-\bar{S}_\delta(s)}{L(s).s} \left[ \frac{1-\bar{S}_\delta\{s+\lambda+v-\sum\lambda_i\bar{S}_i(s)\}}{s+\lambda+v-\sum\lambda_i\bar{S}_i(s)} (M-j+1)(\lambda')^j \right] \\ & \times \prod_{r=1}^{j-1} \left( \frac{M-r+1}{s+\lambda+v+(M-r)\lambda'-\sum\lambda_i\bar{S}_i(s)} \right) \\ & + (\lambda') \prod_{P=0}^{r-1} \left( \frac{M-P}{s+\lambda+v+M-P-1\lambda'-\sum_{i=1}^N\lambda_i\bar{S}_i(s)} \right) + 1 \Big] \\ & \dots\dots\dots(40) \end{aligned}$$

It may be seen that

$$\begin{aligned} \bar{P}_{up}(s) + \bar{P}_{down}(s) &= \frac{1}{s} \\ \dots\dots\dots(41) \end{aligned}$$

### 2.4 Ergodic Behaviour of the System

Using Abel's Lemma,

$$\lim_{s \rightarrow 0} S\bar{F}(s) = \lim_{t \rightarrow \infty} F(t) = F \text{ (say),}$$

provided the limit on right exist, the system's steady state probabilities are obtained as under :

$$\begin{aligned} P_{0.M} &= \frac{1}{L'(0)} \\ \dots\dots\dots(42) \end{aligned}$$

$$\begin{aligned} P_{0.M-r} &= \frac{M}{(M-r)L'(0)} \\ \dots\dots\dots(43) \end{aligned}$$

$$\begin{aligned} P_R &= \frac{M\lambda' M_j}{L'(0)} \\ \dots\dots\dots(44) \end{aligned}$$

$$\begin{aligned} P_{F_i.M} &= \frac{\lambda_i M_i}{L'(0)} \\ \dots\dots\dots(45) \end{aligned}$$

$$\begin{aligned} P_{F_i.M-r} &= \frac{\lambda_i M_i}{L'(0)} \cdot \frac{M}{M-r} \\ \dots\dots\dots(46) \end{aligned}$$

$$\begin{aligned} Q_{F_i.M-j} &= \frac{\lambda_i M_i . M \lambda' . M_j}{L'(0)} \\ \dots\dots\dots(47) \end{aligned}$$

$$\begin{aligned} \text{and } P_f &= \frac{vM_\delta}{L'(0)} \cdot \left[ M\lambda' M_j + \frac{M}{M-r} \right] \\ \dots\dots\dots(48) \end{aligned}$$

where  $L'(0) = \left[ \frac{d}{dx} L(s) \right]_{s=0}$

### 2.5 Particular Cases

**(A)** Class  $L_1$  consists of  $N$  components in series and Class  $L_2$  consists of  $M$  components in Parallel Redundancy, assuming that the failure of any two components out of  $M$  causes the system to work in reduce efficiency state.

Substituion of  $j = 2$  in relation (32) through (38), one gets

$$\bar{P}_{0.M}(s) = \frac{1}{A(s)} \{s + \lambda + v + \overline{M-1}\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)\} \dots\dots\dots(49)$$

$$\bar{P}_{0.M}(s) = \frac{M\lambda'}{A(s)} \dots\dots\dots(50)$$

$$\bar{P}_R(s) = \frac{M(M-1)(\lambda')^2}{A(s)} \cdot \frac{1 - \bar{S}_2\{s + \lambda + v - \sum \lambda_i \bar{S}_i(s)\}}{s + \lambda + v - \sum \lambda_i \bar{S}_i(s)} \dots\dots\dots(51)$$

$$\bar{P}_{F.M}(s) = \frac{\lambda'}{A(s)} \cdot \frac{1 - \bar{S}_i(s)}{s} \{s + \lambda + v + \overline{M-1}\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)\} \dots\dots\dots(52)$$

$$\bar{P}_{F.M}(s) = \frac{\lambda'}{A(s)} \cdot \frac{1 - \bar{S}_i(s)}{s} M\lambda' \dots\dots\dots(53)$$

$$\bar{Q}_{F.M-2}(s) = \frac{\lambda_i}{A(s)} \cdot \frac{1 - \bar{S}_i(s)}{s} \cdot M(M-1)(\lambda')^2 \frac{1 - \bar{S}_2\{s + \lambda + v - \sum \lambda_i \bar{S}_i(s)\}}{s + \lambda + v - \sum \lambda_i \bar{S}_i(s)} \dots\dots\dots(54)$$

$$\bar{P}_F(s) = \frac{v}{A(s)} \cdot \frac{1 - \bar{S}_\delta(s)}{s} \left[ \frac{1 - \bar{S}_2\{s + \lambda + v - \sum \lambda_i \bar{S}_i(s)\}}{s + \lambda + v - \sum \lambda_i \bar{S}_i(s)} M(M-1)(\lambda')^2 + M\lambda' + \{s + \lambda + v + \overline{M-1}\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s)\} \right] \dots\dots\dots(55)$$

Where,

$$A(s) = \left\{ s + \lambda + v + M\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s) \right\} \left\{ s + \lambda + v + \overline{M-1}\lambda' - \sum_{i=1}^N \lambda_i \bar{S}_i(s) \right\} - M(M-1)(\lambda')^2 \times \bar{S}_2\{s + s + \lambda + v + M\lambda' - \sum_{i=1}^N \bar{S}_i(s) \cdot \lambda_i\} - v \cdot \bar{S}_\delta(s) [M(M-1)(\lambda')^2 \bar{S}_2[s + \lambda + v - \sum_{i=1}^N \bar{S}_i(s) \cdot \lambda_i] + M\lambda + \{s + \lambda + \overline{M-1}\lambda' - \sum \lambda_i \bar{S}_i(s)\}]$$

### **(B) Repair Follows Exponential Time Distribution**

Setting  $\bar{S}_i(s) = \frac{\eta_i}{s + \eta_i}$  and  $\bar{S}_i(s) = \frac{\phi_i}{s + \phi_i}$  and  $\bar{S}_i(s) = \frac{\delta}{s + \delta}$  in relations (32) through (38), one may get

$$\bar{P}_{0.M}(s) = \frac{1}{B(s)} \dots\dots\dots(56)$$

$$\bar{P}_{0.M-r}(s) = \frac{(\lambda')^r}{B(s)} \prod_{P=0}^{r-1} \left( \frac{M-P}{s+\lambda+v+\overline{M-P-1}\lambda' - \sum \frac{\lambda_i \eta_i}{S+\eta_i}} \right) \dots\dots\dots(57)$$

$$\bar{P}_R(s) = \frac{(\lambda')^j(M-j+1)}{\left\{s + \lambda + v + \phi_j - \frac{\lambda_i \eta_i}{S + \eta_i}\right\} \cdot B(s)}$$

$$\times \prod_{r=1}^{j-1} \left[ \frac{M-r+1}{s+\lambda+v+\overline{M-r}\lambda' - \sum \frac{\lambda_i \eta_i}{S+\eta_i}} \right] \dots\dots\dots(58)$$

$$\bar{P}_{F.M}(s) = \frac{\lambda_i}{(S+n_i) \cdot B(s)} \dots\dots\dots(59)$$

$$\bar{P}_{F.M-r}(s) = \frac{\lambda_i(\lambda')^r}{(S+n_i) \cdot B(s)} \prod_{P=0}^{r-1} \left[ \frac{M-P}{s+\lambda+v+\overline{M-P-1}\lambda' - \sum \frac{\lambda_i \eta_i}{S+\eta_i}} \right] \dots\dots\dots(60)$$

$$\bar{Q}_{F.M-j}(s) = \frac{\lambda_i}{(S+n_i) \cdot B(s)} \cdot \frac{(\lambda')^j(M-j+1)}{\left\{s+\lambda+v+\phi_j - \sum \frac{\lambda_i \eta_i}{S+\eta_i}\right\}}$$

$$\times \prod_{r=1}^{j-1} \left( \frac{M-r+1}{s+\lambda+v+\overline{M-r}\lambda' - \sum \frac{\lambda_i \eta_i}{S+\eta_i}} \right) \dots\dots\dots(61)$$

$$\bar{P}_F(s) = \frac{v}{B(s)} \cdot \frac{1}{s+\delta} \left[ \frac{M(M-1)(\lambda')^2}{s+\lambda+v+\phi_j - \sum \frac{\lambda_i \eta_i}{S+\eta_i}} + M\lambda' \right.$$

$$\left. \left\{ s + \lambda + v + \overline{M-1}\lambda' - \sum \frac{\lambda_i \eta_i}{S+\eta_i} \right\} \right] \dots\dots\dots(62)$$

Where,  $B(s) = s + \lambda + v + M\lambda' - \sum \frac{\lambda_i \eta_i}{S+\eta_i} - \frac{(M-j+1)(\lambda')^j \cdot \phi_j}{s+\lambda+v+\phi_j - \sum \frac{\lambda_i \eta_i}{S+\eta_i}}$

$$\times \prod_{r=1}^{j-1} \left( \frac{M-r+1}{s + \lambda + v + \overline{M-r}\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right) - \frac{v\delta}{S + \delta}$$

$$\left[ \frac{(M-j+1)(\lambda')^j \cdot \phi_j}{s + \lambda + v + \phi_j - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \times \prod_{r=1}^{j-1} \left( \frac{M-r+1}{s + \lambda + v + \overline{M-r}\lambda' - \sum \frac{\lambda_i \eta_i}{S + \eta_i}} \right) \right.$$

$$\left. + \prod_{P=0}^{j-1} \frac{(\lambda')^r(M-P)}{s+\lambda+v+\overline{M-P-1}\lambda' - \sum \frac{\lambda_i \eta_i}{S+\eta_i}} + 1 \right]$$

### 2.6 Numerical Computation

To depict the result numerically and graphically,  $P_{up}^{Res.}(t)$  obtained from (39) has been plotted against t in the steady state for  $N = 1, \lambda = 0.03, \lambda' = 0.02, v = 0.01, M = 10, \delta = \eta = \phi = 1, i = 1, j = 2$  for any complex system, we have  $P_{up}^{Res.}(t) = 0.431162287 + 0.005188786 \times$

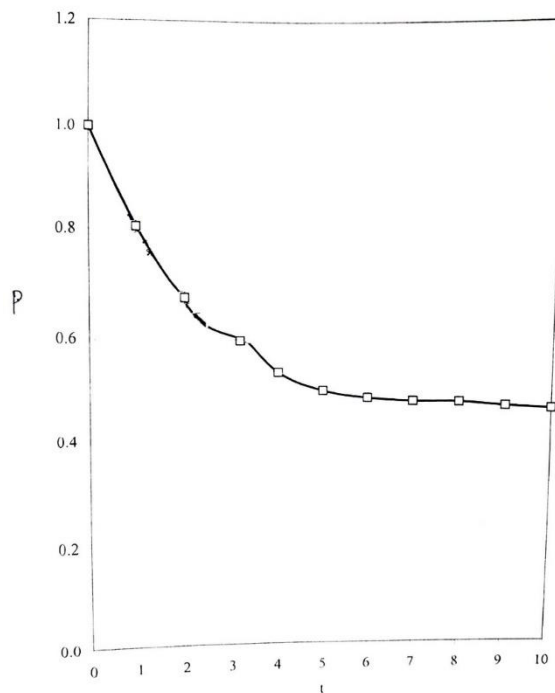
$$e^{1-0.33690t} + 0.087040988 \times e^{1-0.33690t} - 0.05758925 \times e^{-1.160167t} + 0.586153689 \times e^{-0.42864t}$$

$$-0.051956792 \times e^{-0.81451t} \dots\dots(63)$$

Effect of time on  $P_{Up}^{Res.}(t)$

From equation (2.1)

t	P
0	1
1	0.80383269
2	0.675071482
3	0.622997268
4	0.535463433
5	0.499304304
6	0.485653321
7	0.480195324
8	0.480099485
9	0.473523056
10	0.469211126



## Conclusion

From the Table 2.1 and Graph 2.1 We concludes that  $P_{Up}^{Res.}(t)$  of the system decreases with respect of time.

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