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Long Run Availability of a Two Unit Cold Standby Repairable System Subject to Two Types of Critical Errors

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Abstract

In this paper deals a cold stand by system composed of two identical units S_0 (main operating unit) and S_- (standby unit) which is taken into function through a switching mechanism which is not hundred percent reliable. It is obvious that the stand by unit is switched to operate when the operating unit fails and the switching device which is used to put the standby unit into operation may be perfect or imperfect at the time of need operating unit may fail either due to normal or chance so we evaluated various important measures of system effectiveness and reliability parameters.

Keywords

Reliability, MTTF, Repairable, Standby Unit, Cold standby system, Busy time.

Introduction

Several researchers had considered two unit cold standby repairable systems and evaluated various reliability parameters. But they have not taken into account the failure due to software failure or critical human error. In this paper we considered a cold stand by system composed of two identical units S_0 (main operating unit) and S_- (standby unit) which is taken into function through a switching mechanism which is not hundred percent reliable.

After the failure S_0 unit, if the switching mechanism is perfect, then the S_s unit starts functioning. However, If switching mechanism is not perfect, the system goes to failed state till the repair of switching mechanism is carried out. When the S_s unit begins functions, so failed unit undergoes the repair. In all these operative situations. the system may also suffer break-down due to the critical human error or due to some corruption in software of the equipment.

Assumptions

(1) Initially the system is good

(2) Each until has only two states of operation either good or failure.

(3) Only one repair facility repairs all types of failures and after repair the units worked as a new with normal efficiency.

(4) Switching over mechanism is perfect, but subject to environmental conditions and repairable.

(5) All failure follow the exponential time distributions.

(6) Repair rates of the units of standby system and the switching mechanism are constant while the repair of failed states due to critical human error or software failure follow the general time distribution.

Notations

Description of states of the system

$S_{0.5}(t)$ = At any time $'\text{t}'$ both operative and the standby unit are good

$S_{0.5}(t)$ = At any time $'\text{t}'$ main unit is failed but standby unit is in operation.

$S_i(t)$ = At any time $'\text{t}'$ failed state due to failure of switching mechanism.

$S_f(x, t)$ = At any time $'\text{t}'$ failed state due to software failure a failure of standby unit under repair, elapsed repair time is x.

$S_h(y, t)$ = At any time $'\text{t}'$ failed state due to critical human error under repair, elapsed repair time is y.

Symbol

λ	-	failure rate of main unit of standby system
λ_h	-	constant failure rate due to critical human error
λ_s	-	constant failure rate due to software corruption or failure of standby unit
v	-	constant repair rate of units of standby system
w	-	constant repair rate of switching over device.
$p.q$	-	probability of switching mechanism being perfect/imperfect (p+q=1)
ϕ (x)	-	transition repair rate in the state $S_{f.s}$
Ψ(y)	-	transition repair rate in the state S_h

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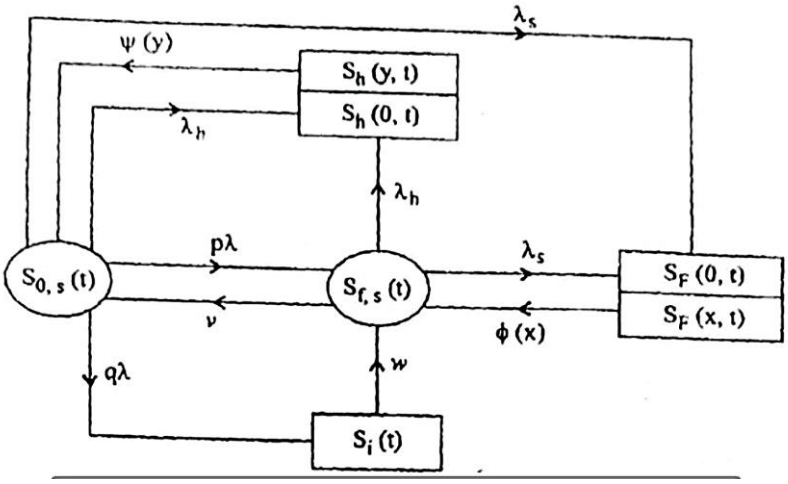
Pictorial Representation of Flow of States



State Probabilities

At any time 't'

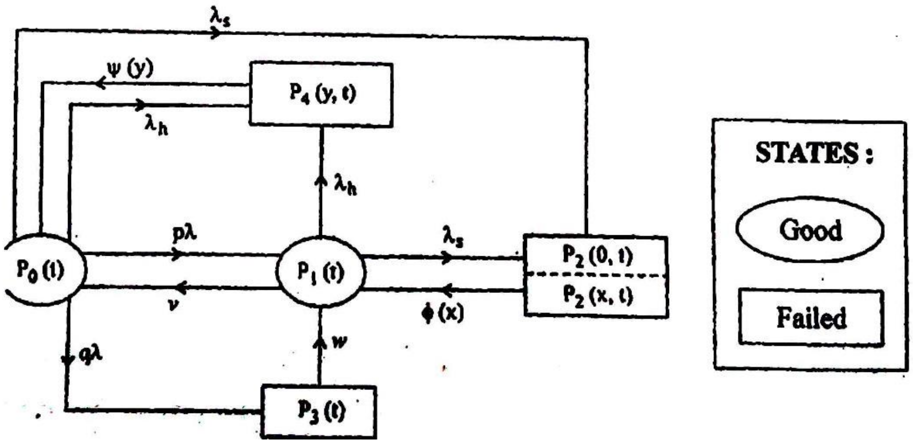
$$P_0(t) - \text{Probability of the system in state } S_0.s(t)$$

$$P_f(t) - \text{Probability of the system in state } S_f.s(t)$$

$$P_i(t) - \text{Probability of the system in state } S_i.s(t)$$

$$P_h(y,t) - \text{Probability of the system in state } S_h s(y,t)$$

$$P_f(x,t) - \text{Probability of the system in state } S_f.s(x,t)$$



Mathematical Model of the Problem

By probability and continuity arguments the difference differential equations for the Stochastic Process, which is continuous in time, discrete in space are as follows:-

$$\left[\frac{\partial}{\partial t} + p\lambda + \lambda_h + q\lambda + \lambda_0\right] P_0(t) = vP_1(t) + \int_0^\infty P_4(y,t)\Psi(y)dy \quad ....(1)$$

$$\left[\frac{d}{dt} + \lambda_s + v + \lambda_h\right] P_1(t) = w.P_3(t) + \int_0^\infty P_4(x,t)\phi(x)dx \quad ....(2)$$

$$\left[\frac{\partial}{\partial x} + \frac{\partial}{\partial x} + \phi(x)\right] P_2(x,t) = 0 \quad ....(3)$$

$$\left[\frac{\partial}{\partial t} + w\right] P_3(t) = q\,\lambda.P_0(t) \quad ....(4)$$

$$\left[\frac{\partial}{\partial y} + \frac{\partial}{\partial t} + \Psi(y)\right] P_4(y,t) = 0 \quad ....(5)$$

Boundary Conditions

$$P_2(0,t) = \lambda_s\{P_1(t) + P_0(s)\} \quad ....(6)$$

$$P_4(0,t) = \lambda_h\{P_1(t) + P_0(s)\} \quad ....(7)$$

Initial Condition

$$P_k(t) = f(x) = \begin{cases} -1, & If\ k = 0 \\ 0, & otherwise \end{cases} \quad ....(8)$$

Solution of the Mathematical Model

Take Laplace Transforms of equation (1) to (7) and using (8) one may obtain following equations:

$$(s + p\lambda + \lambda_h + q\lambda + \lambda_0)\overline{P_0}(s) = 1 + v.\bar{P}_1(s) + \int_0^\infty \bar{P}_4(y,s)\,\Psi(y)dy \quad ....(9)$$

Pictorial Representation of Flow of States

$$s + \lambda_s + v + \lambda_h)\bar{P}_1(s) = w.\bar{P}_2(s) + \int_0^0 \bar{P}_3(x,s)\phi(x)dx. \quad ....(10)$$

$$\left[s + \frac{\partial}{\partial x} + \phi(x)\right] \bar{P}_2(x,s) = 0 \quad ....(11)$$

$$(s + w)\bar{P}_3(s) = q\lambda\bar{P}_0(s) \quad ....(12)$$

$$\left[s + \frac{\partial}{\partial x} + \Psi(y)\right] \bar{P}_4(y,s) = 0 \quad ....(13)$$

$$\bar{P}_2(0,s) = \lambda_s\{\bar{P}_1(s) + \bar{P}_0(s)\} \quad ....(14)$$

$$\bar{P}_4(0,s) = \lambda_h\{\bar{P}_1(s) + \bar{P}_0(s)\} \quad ....(15)$$

Now solving equation (11), by using (14), we obtain

$$\bar{P}_2(x,s) = \lambda_h\{\bar{P}_1(s) + \bar{P}_0(s)\}e^{-s,x-\int_0^x \phi(x)dx} \quad ....(16)$$

$$\bar{P}_2\,(s) = F_\phi(s)\lambda_s\{\bar{P}_1(s) + \bar{P}_0(s)\} \quad ....(17)$$

From equation (12) and by using (15), we obtain

$$\bar{P}_3(s) = \frac{q\lambda\bar{P}_0(s)}{(s+w)} \qquad ....(18)$$

$$\bar{P}_4(y,s) = \lambda_h\{\bar{P}_1(s) + \bar{P}_0(s)\}e^{-sy-\int_0^y \Psi(y)dy} \qquad ....(19)$$

$$\bar{P}_s(y,s) = \{\bar{P}_1(s) + \bar{P}_0(s)\}F_\Psi(s) \qquad ....(20)$$

Now solving equation (10) and using (16) we obtain

$$\bar{P}_1(s) = \bar{P}_0(s).k \qquad ....(21)$$

Now solving equation (9) and using (19), we obtain

$$\bar{P}_0(s) = \frac{1}{A(s)} \qquad ....(22)$$

Where A(s) = [s + $p\lambda + \lambda_h + q\lambda + \lambda_s - v.K - \lambda_h\ (1+K)\ F_\Psi(s)$]

$$K = \frac{(wq\lambda+\lambda_3)}{(s+w)[\ s+\lambda_s+v+\lambda_h-\lambda_s\bar{S}_\phi(s)]}$$

Now equations (17) to (21) yield

$$\bar{P}_1(s) = \frac{K}{A(s)}$$

$$\bar{P}_2(s) = \lambda_s F_\phi(s)\frac{(K+1)}{A(s)} \qquad ....(23)$$

$$\bar{P}_3(s) = \frac{q\lambda}{(s+w)}\frac{K}{A(s)} \qquad ....(24)$$

$$\bar{P}_4(s) = \frac{\lambda_s(1+K)F_\Psi(s)}{A(s)} \qquad ....(25)$$

It is interesting to note that

$$\sum_{1=0}^{4}\bar{P}_i(s) = \frac{1}{s} \qquad ....(26)$$

Ergodic Behavior of the System

By using Abel's Lemma, i.e.,

$$\lim_{s\to 0}\bar{F}(s) = \lim_{t\to\infty}F(t) = F\ (say)$$

Provided R.H.S. Limit exists, one may obtain the time independent probabilities of the system being in various states as follows :

$$P_0 = \frac{1}{A'(0)} \qquad ....(27)$$

$$P_1 = \frac{H}{A'(0)} \qquad ....(28)$$

$$P_2 = \lambda_s F_\phi\frac{(h+1)}{A(0)} \qquad ....(29)$$

$$P_3 = \frac{q^\lambda}{w}\frac{H}{A'(0)} \qquad ....(30)$$

$P_4 = \lambda_\delta \frac{(1+H)}{A'(0)} F\Psi$ ....(31)

Where $H = \frac{(wq^{\lambda}+\lambda_s)}{w\,[\lambda_s+v+\lambda_h-\lambda\bar{S}_v(0)]}$

Particular Cases

When repairs follow exponential time distribution

Setting $\bar{S}_y(s) = \frac{\gamma}{s+\gamma}$. where $\gamma = \phi, \Psi$, equation (22) to (25) yield

$\bar{P}_0(s) = \frac{1}{B(s)}$ ....(32)

$\bar{P}_1(s) = \frac{T}{B(s)}$ ....(33)

$\bar{P}_2(s) = \lambda_s \frac{(1+T)}{(s+\phi)B\,(s)}$ ....(34)

$\bar{P}_3(s) = \frac{q^{\lambda}}{(s+w)} \frac{(T)}{B\,(s)}$ ....(35)

$\bar{P}_4(s) = \frac{\lambda_h(1+T)}{B\,(s)(s+\Psi)}$ ....(36)

Where $B(s) = \left[s + p\lambda + \lambda_h + q\lambda + \lambda_s - v.T - (1-T)\lambda_h \frac{1}{(s+\Psi)}\right]$

and $T = \frac{(s+\Psi)(w.q\lambda+\lambda_s)}{(s+w)(s+\Psi)(s+\lambda_s+v+\lambda_h)-\lambda_s\Psi]}$

Evaluation of Up and Down State Probabilities

For the non-repairable system, i.e, when all repair rates are zero then one may obtain

$\bar{P}_{up}(s) = \frac{(1+T_1)}{B_1(s)}$ ....(37)

and $\bar{P}_{down}\,(s) = \frac{1}{(s)} - \bar{P}_{up}(s)$

Where $B_1(s) = \left[s + p\lambda + \lambda_h + q\lambda + \lambda_s(1-T_t)\frac{\lambda_h}{s}\right]$

and $T_t = \frac{\lambda_s}{s(s+\lambda_s+\lambda_h)}$

Numerical Illustration

When repair is not feasible one may obtain

$R(t)\ 0 = \exp\{-(p\lambda + \lambda_h + q\lambda + \lambda_s)t\}$ ....(38)

M.T.S.F. (Mean Time to System Failure)

$= \int_0^{\infty} R(t)dt$

$= \frac{1}{(p\lambda+\lambda_h+q\lambda+\lambda_s)}$ ....(39)

Effect λ_s on R(t)

Setting the value $\lambda = 0.01$, p = 0.5, $\lambda_s = 0.02$, t =1, q = 0.5 in equation (38)

Table -1

λ_s	R(t)
0.03	0.9417645336
0.04	0.9323938199
0.05	0.9231163464
0.06	0.9139311853
0.07	0.9048374180
0.08	0.8958341353
0.09	0.8869204367
0.10	0.8780954309

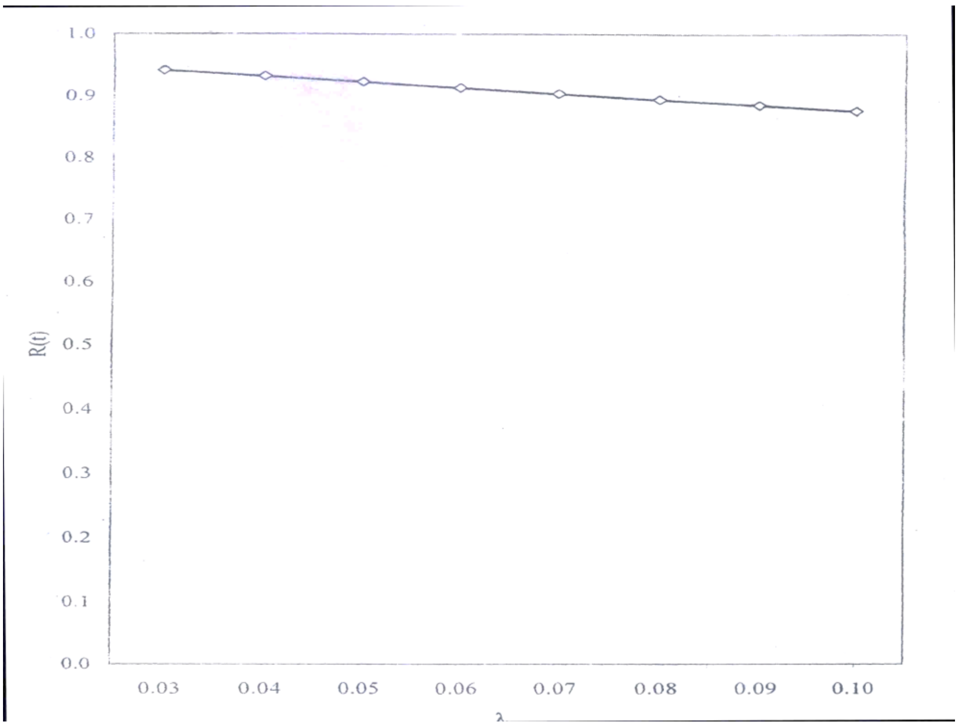
Effect of time on Reliability

Setting the value $\lambda = 0.01$, p = 0.1, p = 0.05, $\lambda_h = 0.02$, q = 0.5 $\lambda_s = 0.003$ in equation (38)

in equation (38)

Table-2

t	R(t)



0	1.0000000000
1	0.9748223790
2	0.9502786705
3	0.9263529143
4	0.9030295517
5	0.8802934158
6	0.8581297218
7	0.8365240569
8	0.8154623712
9	0.7949309686
10	0.7749164980

GRAPH-1

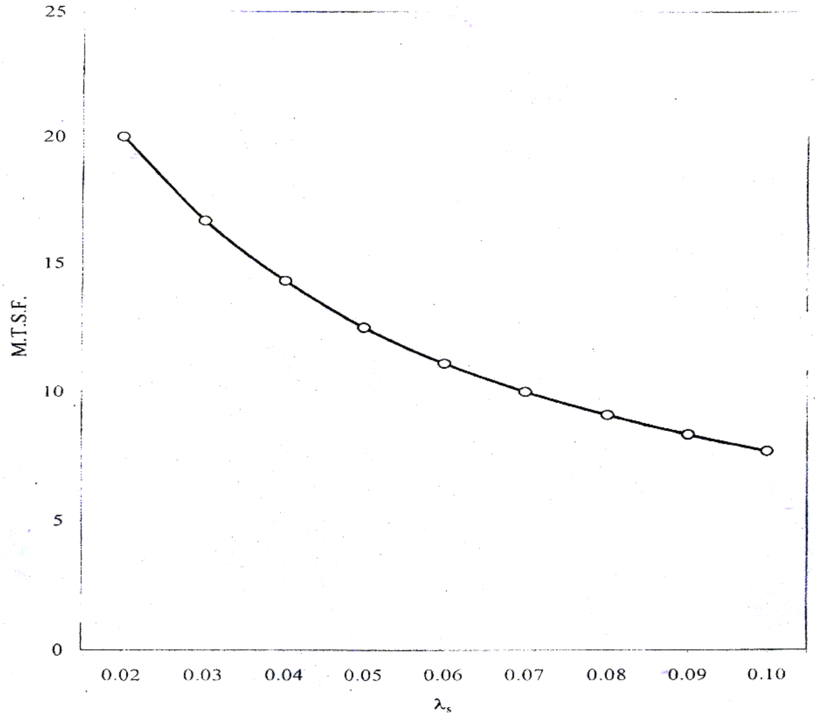
Effect of λ_∞ on the M.T.S.F.

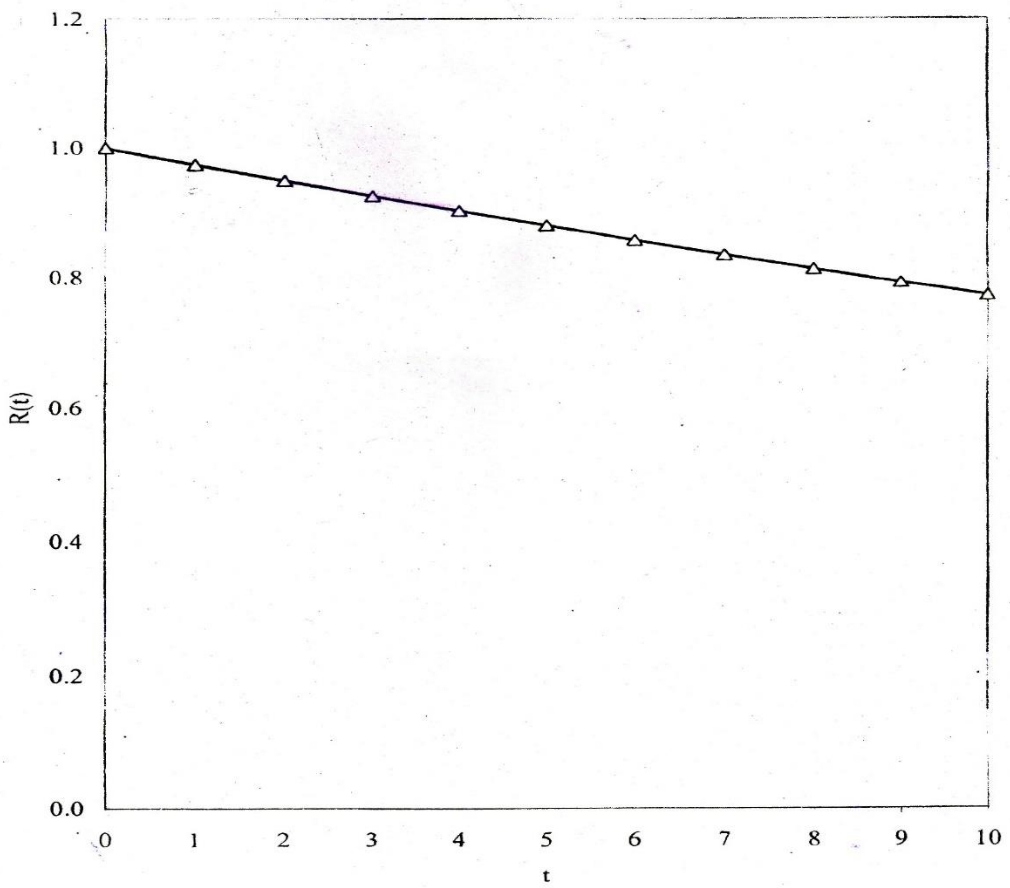
Setting the value $\boldsymbol{\lambda} = 0.01$, $\lambda_h = 0.02$, $\mathrm{p} = 0.5, \mathrm{q} = 0.5$ in equation (39)

Table -3

λ_h	M.T.S.F.
0.02	20.00000000
0.03	16.66666666
0.04	14.28571429
0.05	12.50000000
0.06	11.11111100
0.07	10.00000000
0.08	9.090909091
0.09	8.333333333
0.10	7.692307692

GRAPH-2


25
20
15
10
5
0
M.T.S.F.
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.10
λs


1.2
1.0
0.8
0.6
0.4
0.2
0.0
R(t)
0
1
2
3
4
5
6
7
8
9
10
t

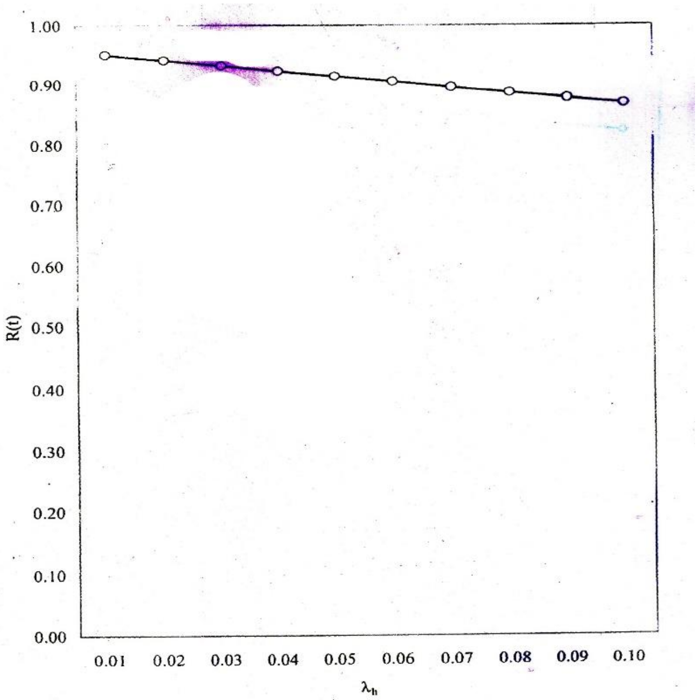
GRAPH-3

Effect of λ_h on Reliability

Setting the value t =1, $\lambda = 0.01$, p = q = 0.5 $\lambda_s = 0.003$ in equation (35)

Table-4

λ_h	R (t)
0.01	0.9512294245
0.02	0.9417645336
0.03	0.9323938199
0.04	0.9231163464
0.05	0.9139311893
0.06	0.9048374180
0.07	0.8958341353
0.08	0.8869204367
0.09	0.8780954309
0.10	0.8693582354



GRAPH-4

Conclusion

From the graph and data we conclude that reliability of the system decreases with the increases in time software failure rate, in human failure rate but remains high for sufficient long interval of time. Also MTSF falls rapidly as $\lambda_s$increases

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